Riccardo Fiorito[1]

Government Debt, Taxes and Growth

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Abstract - By using a simple discrete-time model we evaluate the impact of distortionary taxation on the government debt-to-gdp ratio. Once the standard model is modified accordingly, it appears that rising taxes has a growth cost which increases as long as the debt-to-gdp ratio rises. The empirical implementation uses data drawn from recent Italy's record and is based on realistic shocks to the relevant parameters. A major finding is the importance of the debt level - not only of the dynamics - to stabilize the debt-to-gdp ratio. A second outcome of the paper is showing that sustainable tax rates are remarkably lower than those prevailing in Italy since the 80s.

JEL classification codes - H2, H6, 04
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1. Introduction

As recently stated "neither economic theory nor history offers much guide to what is a sustainable level of public debt. In the end it is the investors who decide, by demanding much higher interest on bonds" (The Economist, 01/22/00). In this vein, the specialized literature has shown that what matters for debt sustainability is not exceeding a particular debt-to-gdp ratio, it rather being important that the rate of growth of the economy exceeds the real cost of the debt. The main weakness of the approach is that both the rate of growth and the cost of debt are exogenous. A second weakness is not considering that debt financing through taxes can reduce the rate of growth of the economy, then affecting the debt-to-gdp ratio. The suggested formulation stresses that financing debt through taxation has a growth cost which is neglected in the debt sustainability literature and which can distort the assessment of the time span required to reach any given debt-to-gdp ratio.

Here, we try to combine these two different streams of literature by introducing into the government budget constraint a simple distortionary taxation scheme and by allowing the growth of the economy to reflect the burden of taxation. Conversely, we neither analyze the potential effects of a rising government debt on its cost nor the implications of inserting money into the budget constraint. For the sake of simplicity, lump-sum taxes are also ignored.

[1]Department of Economics, Piazza San Francesco 7, 53100 Siena, Italy (e/mail:<rfiorito@iol.it>). I am indebted to Riccardo Bonci for excellent research assistance and to Flavio Casprini, Donald D.Hester, Giancarlo Marini, John Seater and Salvatore Vinci for useful comments and suggestions. Likewise, I am also grateful to the participants at a seminar at the Istituto Navale of Naples and at the II Siena-LSE Workshop (Siena, April 2000). CNR funding is acknowledged.

In Section 2 we report and discuss a few historical data on Italy's debt determinants. In section 3 we present a toy model and its main implications for the debt-to-gdp dynamics. To add more realism, we solve it for parameters close to the actual data and we provide some sensitivity analysis to evaluate how deeply simulations react to the tax rate shifts. Section 4 shortly discusses the relation between sustainable tax rates and the level of the debt-to-gdp ratio. Section 5 concludes.

2. Some reference data

In this paragraph we report a few reference data on Italy's debt determinants over the last 25 years. For convenience, we divide the full sample into five-year subperiods that we tentatively characterize as follows:

65-69: a growing economy and a sound fiscal policy
70-74: concomitant labor and oil shocks
75-79: responses to the supply shocks
80-84: rising worldwide interest rates
85-89: taming inflation and debt explosion
90-94: yet deficits are still high..
95-98: fiscal discipline in the EMU perspective.

Of course, different labels and different periods can be used. What is important is noting that each interval contains both recession and expansion phases that should make period averages relatively independent of the business cycle.

Table 1: Components of the government debt-to-gdp ratio

years	b^*	τ	g^*	r	π	σ	def/y
65-69	32.4	29.3	31.1	4.1	3.0	5.5	-3.5
70-74	39.1	29.0	33.0	5.1	10.9	4.1	-6.6
75-79	53.8	30.7	34.4	7.6	16.5	3.2	-10.1
80-84	62.5	36.2	38.4	10.6	16.7	1.6	-10.6
85-89	87.5	39.8	39.0	9.3	7.2	3.1	-11.0
90-94	98.0	44.6	41.0	11.1	5.6	1.0	-9.9
95-98	109.0	46.6	39.1	9.1	3.8	1.6	-4.9

(Source: Oecd, Analytical Database)

The variables reported in the Table 1 disentangle between revenues and spending, hence widening the typical decomposition of the government debt changes (Buiter, 1985): namely, they denote the debt-to-gdp ratio (b^*) and the effective tax rate (τ) which is simply obtained as the ratio between total receipts and nominal gdp. Then, we report the primary spending-to-gdp ratio (g^*) and the average cost of debt (r). The latter is obtained as the ratio between the net interest government spending and the outstanding debt level. The gdp deflator inflation rate (π) helps obtaining a proxy for the ex-post real cost of debt to be compared with the real gdp growth (σ)[2]. Finally, we report the ratio between total government balances and nominal gdp (def/y).

Table 1 shows the value of some critical parameters for understanding the sources of the debt creation in Italy since the mid-60s. In this respect, a few comments are in order:

- the effective tax rate τ and the debt-to-gdp ratios are the only variables that increase monotonically over the sample.[3]

- the success in curbing inflation in the second half of the 80s revealed a debt issue that was masked in the previous decade by high inflation, partially offsetting the occurrence of huge government deficits.

- the growth reduction in the 90s came along with higher real interest rates. The real cost of debt was even higher since the nominal cost adjusted slowly to the disinflation, presumably because the debt management policy aimed at lengthening the debt maturity.

- the reported data show that the last fiscal adjustment occurred by rising taxes but was also made by reducing in about the same proportion the primary spending to gdp ratio.

3. The model

The model assumes that real GDP growth is not exogenously given but is a negative function of the aggregate tax rate. This can be interpreted as a reduced form approximation of a production function where both capital and labor inputs reflect the burden of distortionary taxation.

Actually, in the neoclassical growth model taxation should not influence the rate of growth in the long run because the capital output ratio must be constant in the steady state. Hence, the distortionary effect on capital formation acts during the transition to the steady state but vanishes afterwards. This conclusion does not hold in the endogenous growth model where the constant return and the decreasing marginal productivity assumptions are replaced by increasing returns. The major implication is that rising tax rates reduces the rate of growth also in the long run because the incentives to accumulate human or physical capital are also reduced (Easterly-Rebelo,1993).

The approach we follow here is mostly policy-oriented and does not address directly any of the fundamental disputes in growth theory (Barro-Sala-i-Martin, 1995) but is based on a reduced form conjecture that the rate of growth of the

[2] To simplify the notation, we do not make here the distinction between σ and σ_t which is crucial for the next section model.

[3] However, by taking annual data instead of five-year averages, the debt-to-gdp ratio rises until 1994 and falls afterwards.

economy falls after increasing the tax rates. Yet, this conjecture has some empirical strength that we deem useful to exploit for the problem at stake.

For empirical purposes, we did not construct the statutory tax rates which typically include exemptions and contingencies that are difficult to aggregate in each period and even more so in a time-series setting (Barro-Sahasakul,1983). We used instead effective tax rates calculated as the share of revenues with respect to the nominal GDP.[4] This share includes social security receipts that are important for labor allocation and for financing social security on a pay-as-you-go basis.

Let the debt-to-GDP ratio evolve in discrete time units according to the following model where capital letters denote nominal values and lower-case letters denote volumes, respectively:

(1) $B_t = (1 + r_{t-1})B_{t-1} - F_t$

(2) $F_t = T_t - G_t$

(3) $T_t = \tau_t Y_t$

(4) $G_t = p_t g_t$

(5) $g_t = (1 + \theta)g_{t-1}$

(6) $y_t = (1 + \sigma_t)y_{t-1}$

(7) $\sigma_t = \sigma - \alpha\tau_t$.

Equation (1) describes the government debt transition in which B is the outstanding debt value and F is the primary surplus, defined in equation (2) as the difference between total receipts (3) and the primary government spending (4). Revenues are proportional to the nominal gdp and depend on a time-variable and policy-controlled tax rate τ.

To be simple, we assume that the cost of debt (r) refers to one period only. Equations (5)-(6) denote the transition of the primary government spending (5) and of the gdp volumes (6), respectively. The primary spending volume grows at a constant rate which is decided by the policy maker. Conversely, the real gdp growth (7) is not constant but is negatively related to the overall tax rate.

Finally, the following definitions apply:

(8) $b_t^* \equiv B_t/Y_t$

(9) $g_t^* \equiv G_t/Y_t$

[4] Quarterly estimates of consumption, capital and labor tax rates are provided for six major OECD countries by Fiorito and Padrini (2000).

(10) $f_t^* \equiv F_t/Y_t$

(11) $1 + \pi_t \equiv p_t/p_{t-1}$.

Equations (8)-(10) denote the debt, the primary spending and the primary surplus-to-gdp ratio, respectively, while definition (11) denotes the one-period inflation rate. All parameters are positive. All quantities and prices are non-negative.

The basic feature of this model is that the growth rate is negatively affected in equation (7) by the effective tax rate τ. This implies that the unconditional growth rate σ is replaced by a time-varying drift (σ_t) because of taxation.

Another distinctive feature of the model is that government spending decisions reflect quantities only because prices are out of government control. Since our goal is showing how the sustainable debt path changes after inserting distortionary taxation only, prices are exogenous as in the standard case: hence, neither money stock enters the model nor prices reflect the fiscal solvency condition that the real value of government liabilities equates the present value of the future primary surplus.[5]

As we mentioned before, equation (6) can be understood as a transformation of any production function $y_t = A_t F(k_t, n_t)$ where A_t is a shifting, nearly integrated, technology parameter characterized by long-lasting memory. Hence, the following labor (n) and capital (k) demand equations are obtained by maximizing a profit function whose costs include tax rates on labor (τ_t^n) and capital (τ_t^k)

(12) $n_t = n(w_t, w_{t-1}.., r_t, r_{t-1}.., \tau_t^n, \tau_{t-1}^n.., \tau_t^k, \tau_{t-1}^k, n_{t-1},..)$

(13) $k_t = k(w_t, w_{t-1}.., r_t, r_{t-1}.., \tau_t^n, \tau_{t-1}^n.., \tau_t^k, \tau_{t-1}^k, k_{t-1},..)$.

The total factor productivity variable (A_t) is omitted in the equations (12)-(13) since the other regressors - exploiting a common factor restriction - include a sufficient number of lags to capture the serial correlation in the technology shock. This allows a dynamic formulation which accounts for adjustment costs and depends in the short run on separate tax rates and before-tax costs for capital (r) and labor (w), respectively.

Accordingly, the resulting indirect production function will be

(14) $y_t = A_t f(w_t, r_t, \tau_t^n, \tau_t^k)$

or else

[5] The fiscal theory of prices dates back to the work of Sims (1994) and Woodford (1995). For an interesting development, see Cochrane (1998).

(15) $y_t = A_t j(\tau_t)$

once i) labor and capital taxation can be nested in an aggregate tax rate equally distorting each source of income and ii) the ratio between net labor and capital costs is approximately constant ($w_t/r_t \cong w/r$). Finally, if tax rates are constant as required by tax smoothing, the growth rate of the economy is the same as the growth rate of the total factor productivity.

Equation (6) is a linear characterization of equation (15) which allows a fiscal feedback on the debt-to-gdp ratio that is generally ignored in the sustainable debt literature. However, there is a difference between these two formulations: since equation (15) is expressed in levels, the corresponding equation for growth should imply a marginal rather than an average tax rate variable (Koester-Kormendi,1989). Yet, we allowed the average tax rate to enter equation (7) because this formulation helps finding a reduced form for the debt transition that can be easily compared with the standard case. A second reason is that average and the marginal tax rates are strongly correlated as shown by the OECD estimates recently used by Nickell and Layard (1999).

Since only one price (p) is assumed, it is immaterial whether ratios (8)-(10) are in nominal or in real terms. Hence, the reduced form for debt and primary surplus ratios to gdp are the following:

(16) $$b_t^* = \frac{1 + r_{t-1}}{(1 + \pi_t)(1 + \sigma - \alpha\tau_t)} b_{t-1}^* - \tau_t + \frac{1 + \theta}{1 + \sigma - \alpha\tau_t} g_{t-1}^*$$

(17) $$f_t^* = \tau_t - \frac{1 + \theta}{1 + \sigma - \alpha\tau_t} g_{t-1}^*.$$

For t≥0, the equation (16) has the following closed-form solution

(18) $$b_t^* = \rho^t b_0^* - (\frac{1 - \rho^t L^t}{1 - \rho L})\tau_t + (\frac{1 - \rho^t L^t}{1 - \rho L})\lambda_t g_{t-1}^*,$$

where $L^j x_t \equiv x_{t-j}$ is the lag operator for any integer j and where $| \rho | \equiv \frac{1+r}{(1+\pi)(1+\sigma-\alpha\tau)} < 1$ and $\lambda_t \equiv \frac{1+\theta}{1+\sigma-\alpha\tau_t}$. The parameter ρ is treated in equation (18) as a constant to avoid cumbersome notation and to simplify the evaluation of partial derivatives. In a stochastic setting, this amounts to assume that the real cost of debt and the real gdp growth are cointegrated and fluctuate around some constant value.

In the long run, $\lim_{t\to\infty} \rho^t = 0$ and equation (18) becomes:

(19) $$b^{**} = -(\frac{1}{1 - \rho})\tau^{**} + (\frac{1}{1 - \rho})g^{**},$$

where the double-starred symbols denote steady-state values.

What should be noted is that equation (16) amounts to the traditional debt transition equation when $\alpha = 0$. In such a case, the debt derivative $\partial b^*/\partial\tau = -1$, which is a constant and higher (absolute) value than those found when the tax rate affects the growth of the economy.

In the more general case ($\alpha \neq 0$), equation (20) shows that the value of the derivative is not constant since three additional variable components are involved: the autoregressive parameter which includes the growth effect of taxation, the past level of the debt-to-gdp ratio and the current or the past level of the primary spending ratio:

$$(20)\ \partial b_t^*/\partial\tau_t = \frac{\alpha(1+r_{t-1})}{(1+\pi_t)(1+\sigma-\alpha\tau_t)^2} b_{t-1}^* + \frac{\alpha(1+\theta)}{(1+\sigma-\alpha\tau_t)^2} g_{t-1}^* - 1.$$

Namely, the response to the tax rate shift will be smaller in absolute value than in the standard case because of the offsetting real gdp response. Secondly, the tax rate impact on the debt-to-gdp ratio will be smaller for higher debt and spending levels since the two fractions in equation (20) are surely positive.

The major economic policy implication is that reducing the government debt burden by taxation is less promising than it appears in the traditional debt model. This is especially true the higher the debt burden is so that spending cuts (or privatizations) should be preferred in these cases.

Conversely, the primary surplus derivative with respect to the overall tax rate τ is

$$(21)\ \ \partial f_t^*/\partial\tau_t = 1 - \frac{\alpha(1+\theta)}{(1+\sigma-\alpha\tau)^2} g_{t-1}^*,$$

which in general is positive and smaller than 1 unless α=0. Once more, the primary surplus response to a tax rate change is not constant, unless $\alpha = 0$ or g^*=0. When this happens $\partial f_t^*/\partial\tau_t = 1$, as in the standard case.[6]

Another implication of the equations (20)-(21) is that second derivatives will exist only for $\alpha \neq 0$. Hence, distortionary taxation will also imply the following expressions

$$(22)\ \partial^2 b_t^*/\partial\tau_t^2 = \frac{2\alpha^2(1+r_{t-1})}{(1+\pi_t)(1+\sigma-\alpha\tau_t)^3} b_{t-1}^* + \frac{2\alpha^2(1+\theta)}{(1+\sigma-\alpha\tau_t)^3} g_{t-1}^* > 0$$

$$(23)\ \ \partial^2 f_t^*/\partial\tau_t^2 = -\frac{2\alpha^2(1+\theta)}{(1+\sigma-\alpha\tau)^3} g_{t-1}^* < 0,$$

whose positive and negative signs conform to the theoretical expectations.

[6] Of course, equations (20) and (21) are obtained by assuming that α and θ are independent policy parameters. Clearly, this is not too realistic, though this issue could be dealt only by adding to the model a number of policy rules that would shift its focus beyond our intentions.

More generally, inserting distortionary taxation into the government budget constraint implies that increasing taxes or decreasing government spending has not the same effect on the debt and on the deficit ratios to gdp as one can immediately notice by evaluating equations (16) and (20). Finally, it seems that accounting for distortionary tax rate does not simply provide more realism but helps widening the sustainable debt algebra to the inherited stock level without the need of resorting to unknown critical thresholds or to arbitrary regime switches.

3.1 Empirics

Because of the (mild) nonlinear feature of the model, we use simulation to figure out how the endogenous variables respond to the same tax rate shock at different values of the debt-to-gdp ratio. To do so, we fixed r = .05, $\pi = .02$, while estimating σ and α in the equation (7bis). For the effective tax rate parameter we used $\tau = .45$, since this is the value found in Table 1 for the late 90s.

We must point at the outset that the reported estimate is not meant to analyze the growth sources in Italy but is simply introduced to obtain numerical values for the parameters in equation (7). To do so, we used annual data, ranging from 1970 to 1998. Further, to account for the possible endogeneity implied by our measure of the effective tax rate τ, we used instrumental variables techniques. After some experimentation, the selected instruments were the past value of the overall tax rate and of the government spending-to-gdp ratio.

Despite the specification is quite crude, results seem satisfactory: parameters are robust to sample changes and are truly statistically significant since the estimated residuals are white, even without inserting lags or adjustment variables. This estimate basically indicates that, reducing by seven percentage points the overall tax rate, about one percent of additional growth could be achieved.

$$\text{(7bis)} \quad \sigma_t = \underset{(.02)}{.076} - \underset{(.05)}{.136}\tau_t, \quad \text{Ljung-Box}(7) = 8.54; \quad \text{see} = .018.$$

In Table 2 we evaluate, in particular, how the debt and the primary deficit ratios respond to a unit tax rate increase. Most importantly, we evaluate how sensitive these responses are at different debt-to-gdp ratios.

The simulations display two interesting results:

i) the higher is the debt-to-gdp ratio, the lesser is the benefit related to rising tax rates

ii) for a given debt-to-gdp ratio, the effect of rising tax rates does not seem too sensitive to the share and to the rate of growth of primary government spending.

Table 2: Debt and primary surplus responses to a tax rate increase

b*	$\partial b^*/\partial\tau^*$	$\partial f^*/\partial\tau^*$	$\partial b^*/\partial\tau^*$	$\partial f^*/\partial\tau^*$	$\partial b^*/\partial\tau^*$	$\partial f/^*\partial\tau^*$
1.2	-.78	.95	-.79	.95	-.79	.96
1.0	-.81	.95	-.82	.95	-.82	.96
0.6	-.86	.95	-.87	.95	-.88	.96
	(g^*=.40; $\theta = 0$)		(g^*=.35; $\theta = 0$)		(g^*=.30; $\theta = 0$)	
b*	$\partial b^*/\partial\tau^*$	$\partial f^*/\partial\tau^*$	$\partial b^*/\partial\tau^*$	$\partial f^*/\partial\tau^*$	$\partial b^*/\partial\tau^*$	$\partial f/^*\partial\tau^*$
1.2	-.78	.95	-.79	.95	-.79	.96
1.0	-.81	.95	-.81	.95	-.82	.96
0.6	-.86	.95	-.88	.95	-.88	.96
	(g^*=.40; $\theta = 0.01$)		(g^*=.35; $\theta = 0.01$)		(g^*=.30; $\theta = 0.01$)	

The main implication of the multipliers shown in Table 2 is that- especially for those countries such as Italy where the debt-to-gdp ratio is among the highest - a fiscal consolidation based on rising revenues is inappropriate not only in terms of the probability of a succesful adjustment (Alesina and Perotti, 1992) but also in terms of the lower speed that high ratios require to get smaller.

It is true that this model is too simple to evaluate possible links between rising revenues and financing expansionary government spending. Yet, a previous detailed study on the stylized facts of government finance in the G-7 (Fiorito, 1997) shows that in most cases government spending does not activate the economy before shocks materialize. On the contrary, most comovements between real government spending variables and gdp rather conform to a passive policy behavior, i.e. to spending components that are lagging and countercyclical instead of leading procyclically the activity level.

4.Sustainable tax rates

We solve the debt transition equation to find which is the tax rate ensuring a sustainable (constant) debt-to-gdp ratio in a growing economy[7]. By defining the ex-post real interest rate i as

$$i_t = r_{t-1} - \pi_t,$$

the parameter $\rho_t \equiv (1 + i_t + \pi_t)/(1 + \pi_t)(1 + \sigma_t)$ for reasonable, small, values of its components can be approximated as $\rho_t \simeq (1 + i_t)/(1 + \sigma_t)$. Thus, the tax rate that stabilizes any value of b* is

$$(24)\ \tau_t = \frac{g_t + b_t^*(i_t - \sigma_t)}{1 + \sigma_t},$$

[7] For a number of suggested indicators, see Blanchard (1993).

where the term in parenthesis denotes the difference between the real cost of debt and the growth of the economy. Remembering that the rate of growth of the real gdp is not a constant, it is apparent that the expression (24) corresponds to a quadratic equation

$$(25)\quad -\alpha\tau_t^2 + (1 + \sigma - \alpha b_t^*)\tau_t - [(g_t + b_t^*(i_t - \sigma)] = 0,$$

where the value of the tax rate is not independent of the debt-to-gdp ratio. Generally, i.e. for $\alpha \neq 0$, there is no reason that $i_t = \sigma$ since σ should be interpreted as the rate of growth characterizing an economy without distortionary taxation. In this respect, if one posits the condition $\sigma_t = i_t$, it follows that $\sigma > i_t$, which is also what we found in the data. This property can be important when differentiating equation (25) with respect to b* since we obtain that the optimal (constant) tax rate value is a positive number

$$(26)\ \tau = \frac{\sigma - i}{\alpha}$$

that for our set of parameters is .34, i.e. a number which is much smaller than the tax rate prevailing in Italy since the 80s.

5. Conclusions

We show with a small discrete-time model that financing government debt through distortionary taxation has a growth cost which is neglected in the sustainable debt literature. A related statement is that the time required to reach any debt-to-gdp ratio is underestimated if the real gdp leakage is ignored.

Simulation results based on recent history parameters show that the leakage is bigger when the debt-to-gdp ratio is higher. The major implication is that for high-debt countries fiscal stabilization should rest more on spending reductions, while more balanced policies should be available for countries characterized by smaller debt-to-gdp ratios. Simple calculations can also show that the effective tax rate in Italy is too high not simply for fostering growth but also to stabilize the debt-to-gdp ratio around a lower value.

Indeed, most of these results basically stem from the finding that the past level of the debt is important: not simply because of the interest spending component but also because distortionary taxes reduce both the government deficit and the gdp growth. While assessing the size and the persistence of the relative effect is an empirical issue, recognizing the importance of the debt level might help to reconcile the academic literature with the economic policy concern that some (low) debt-to-gdp ratio should be preferred to others.

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