Loss Aversion and Higher Moments

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March, 2006

Abstract

This paper studies cumulative prospect theory under the assumption of normal inverse Gaussian distributed returns. Such an assumption takes the higher order moments of financial returns distributions into account. Using numerical techniques, and a model calibration to empirical parameter estimates, I find that: (a) prospect theory investors choose mean-variance efficient portfolios, (b) prospective utility generally *decreases* when skewness increases, (c) prospective utility *increases* with kurtosis, (d) the investor's portfolio choice displays large horizon effects, i.e., a larger weight is placed on stocks if the horizon is longer, and (e) when assuming a normal inverse Gaussian returns distribution, the prospect theory investor places a larger weight on stocks, relative to when normality is assumed. This last finding is explained by the extensive kurtosis of financial returns distributions. The results broaden the understanding of prospect theory preferences in portfolio choice problems.

JEL classificaion: G11

Keywords: prospect theory, normal inverse Gaussian distribution, portfolio choice

*Preliminary and incomplete. I would like to thank my supervisors Annika Alexius and Rolf Larsson for valuable guidance throughout the work of this paper. Financial support from *Stiftelsen Bankforskningsinstitutet* is gratefully acknowledged. Send correspondence to: Department of Economics, Uppsala University, Box 513, SE-751 20 Uppsala, Sweden. Phone: +46 18 471 11 29. Fax: +46 18 471 14 78. E-mail: martin.agren@nek.uu.se.

1 Introduction

Behavioral finance has emerged as an alternative approach to financial economics largely because of the difficulties of the traditional paradigm. The most acclaimed behavioral model of individual decision-making under risk is prospect theory, developed by Kahneman and Tversky (1979) and Tversky and Kahneman (1992). Prospect theory investors derive utility different from the traditional using a specific value function, and they weight probabilities subjectively. The value function differs from standard concave utility functions, e.g., power utility, in several aspects. First, utility is derived over changes in wealth relative to a reference point, as opposed to final wealth states. Second, the function is concave in gains, implying risk aversion, but convex in losses, reflecting a risk seeking behavior in the latter domain. Third, losses loom larger than gains do, causing for a kink in the value function at the reference point. This last property, referred to as loss aversion, is related to the concept of 'first-order' risk aversion (Epstein and Zin, 1990), where investors are sensitive to small changes in wealth.

Loss aversion is the feature of prospect theory that has received most attention. It can disentangle the ambiguous endowment effect (Kahneman, Knetsch, and Thaler, 1990), and together with a risk seeking behavior in losses it provides a plausible explanation to the disposition effect (Shefrin and Statman, 1985).[1] Nevertheless, the most influential finance application of prospect theory is presented by Benartzi and Thaler (1995). Loss aversion causes investors to perceive stocks as more risky if evaluated often, since losses are more likely to occur over short time horizons. In a one-period setting, Benartzi and Thaler (1995) show that this "myopic loss aversion" can explain the historical magnitude of equity premium over bonds if evaluated yearly.

Several applications of prospect theory in financial economics have followed Benartzi and Thaler (1995). Shumway (1997) studies the cross-section of stock returns in an equilibrium asset pricing model with loss averse investors. Barberis, Huang, and Santos (2001) generalize Benartzi and Thaler's (1995) work in a multi-period consumption-based general equilibrium setting. Barberis et al. (2001) show that loss aversion alone does not produce a substantial equity premium. To resolve this, they incorporate an investor sensitivity for prior outcomes, causing for a time-varying loss aversion.

This paper is closer related to the portfolio choice of prospect theory in-

[1] The endowment effect refers to the individual tendency to value something more heavily once owned. The disposition effect concerns the disposition of individual investors to sell wining stocks are hold on to losers. Recently, Barberis and Xiong (2006) find that prospect theory predicts the *opposite* to the disposition effect.

vestors. Several authors have shown that, under the assumption of normally distributed returns, mean-variance efficiency and the CAPM still hold when investors have prospect theory preferences (Levy, De Giorgi, and Hens, 2003; Levy and Levy, 2004; Anderson, 2005; Barberis and Huang, 2005). But how does prospect theory relate to the skewness and kurtosis of a portfolio returns distribution? By assuming normality, such higher order moments are ignored, since the normal distribution is fully characterized by its mean and variance. Taken the fact that loss aversion induces an asymmetric preference over gains and losses, the question is relevant. Furthermore, does mean-variance efficiency hold under a more general distributional assumption than normality? The current paper addresses these issues assuming portfolio returns are normal inverse Gaussian (NIG) distributed.

The NIG distribution is presented by Barndorff-Nielsen (1997) in an application to stochastic volatility modeling. It is a four parameter distribution with the desirable property of capturing the higher order moment characteristics of financial returns distributions. Stylized facts about financial returns show that their empirical distribution is skewed and leptokurtic (center-peaked with fat tails), and clearly not normally distributed. Moreover, Eriksson, Forsberg, and Ghysels (2005) present a transformation of the NIG density, which, roughly speaking, allows the probability density function to be parameterized by the first four central moments directly. Such a parameterization implies a straightforward link between prospective utility and central moments. Hence, the NIG distribution is highly suitable for the purpose of this paper.

I consider a single-period risky gamble with an outcome that is NIG distributed. Using the alternative NIG parameterization, I express the prospective utility of this gamble in direct relation to the first four central moments of the outcome distribution.[2] After calibrating the model parameters using empirical estimates, I find that: (a) prospect theory investors choose mean-variance efficient portfolios, (b) prospective utility generally *decreases* when skewness increases, and (c) prospective utility *increases* with kurtosis. The last two results are surprising and new to the literature. They are explained as follows: When skewness increases, the outcome distribution's tail masses shift from the negative to the positive side, which has a positive effect on utility since extreme losses become less probable. However, the center mass moves in the opposite direction to preserve the mean. This second effect makes moderate-sized negative returns more probable, and, on balance,

[2] In this paper, *expected* prospect theory utility is referred to as prospective utility. Probability weighting is not considered for reasons related to the NIG assumption. See Section 4.1.

prospective utility falls following the rise in skewness.

When kurtosis increases, both distributional tail masses increase while the center mass becomes more concentrated around the mean. Although extreme losses become more likely, moderate-sized ones become less likely, and as it turns out this second effect is more effective than the first, causing prospective utility to rise. The results are robust to alternative estimates of model calibration.

Studying the preference over higher moments in portfolio choice is not new. Kraus and Litzenberger (1976) present an unconditional three-moment CAPM, and find that investors with standard concave utility functions display a preference for (positive) skewness. This result is in line with Arditti (1967), who shows that most standard concave utility functions, e.g., logarithmic and power utility, imply a preference for skewness, since they fulfil the condition of non-increasing absolute risk aversion. Harvey and Siddique (2000) expand the conditional CAPM to include coskewness with the market, and find that conditional coskewness is helpful in explaining the cross-section of equity returns. Concerning prospect theory, Ågren (2005) argues that prospective utility is sensitive to the distributional assumptions made on portfolio returns. While Benartzi and Thaler (1995) place a temporal independence assumption on stock returns, Ågren (2005) introduces conditional heteroskedasticity and finds some evidence that stocks are perceived as more risky when return volatility is known to vary over time. This is related to higher moments since conditional heteroskedasticity affects the unconditional returns distribution. Yet, Ågren (2005) does not present a clear relation between prospective utility and higher moments.

Furthermore, this paper analyzes the portfolio choice of prospect theory investors. A comparison is made between the portfolios chosen under the assumption of NIG distributed returns, and under the assumption of normally distributed returns. This makes it possible to distinguish the effects of higher moments, i.e., skewness and kurtosis, on portfolio choice. Such an analysis is interesting since most papers on this topic assume that financial returns are normally distributed, which, generally, is not the case. The results, for one, show that there are large horizon effects, i.e., a larger weight is placed on stocks as the horizon increases. This result has been shown in previous studies and is not new (Aït-Sahalia and Brandt, 2001). For the other, prospect theory investors place a larger weight on stocks under a NIG distributional assumption for stock returns, compared to when normality is assumed. This result, which is new to the literature, is likely related to the large kurtosis of the stock returns distribution. Taking kurtosis, which is attractive for the prospect theory investor, into consideration, makes stocks more appealing.

The remainder of this paper is outlined as follows. Section 2 introduces cumulative prospect theory, and explains how to incorporate a distributional assumption over outcomes. Section 3 presents the NIG distribution in general, as well as in a more useful alternative form. Section 4 analyzes prospective utility as a function of the gamble outcome distribution's mean, variance, skewness, and kurtosis. Section 5 studies the portfolio choice of prospect theory investors under a NIG assumption. Finally, Section 6 concludes.

2 Cumulative prospect theory

Prospect theory is a descriptive approach to modeling individual decision-making under risk. In its original presentation, Kahneman and Tversky (1979) demonstrate a number of individual violations of neoclassical expected utility based on experimental evidence. Prospect theory is presented as an alternative in spirit of these violations.[3] Although successful in many applications, the original version has its drawbacks. For one, utility can be derived from gambles of only two outcomes, and, for the other, the attractive property of first-order stochastic dominance does not hold. Tversky and Kahneman (1992) propose a resolution they call Cumulative prospect theory (CPT), where utility is derived from gambles of any number of outcomes and first-order stochastic dominance applies. The current paper adopts CPT.

Tversky and Kahneman (1992) present the CPT utility of a gamble G with stochastic outcome X as

$$U(G;\theta) = E_w\left[v(X)\right], \tag{1}$$

where $E_w\left[\cdot\right]$ is the unconditional expectations operator under subjective probability weighting (indicated by w), $v(\cdot)$ is a value function, and θ is a vector of parameters. Probability weighting is induced by the function $w(P)$,

$$w(P) = \frac{P^\tau}{(P^\tau + (1-P)^\tau)^{1/\tau}}, \tag{2}$$

which has the whole cumulative distribution function as an argument, and not just simple probabilities. When $\tau < 1$, objective probabilities are distorted, so that small probabilities are over-weighted and large probabilities are weighted less. This feature of CPT originates in the finding that individuals like lottery-type gambles.[4] When $\tau = 1$, the weighting function collapses

[3]Camerer (1998) points out several fruitful applications of prospect theory, not only in finance, but in areas such as labor and macro economics.

[4]A rational individual should not invest in a lottery ticket since the expected value is negative (the probability of wining is miniscule). However, individuals do buy lottery tickets, since they subjectively over-weight the probability of wining.

so that $w(P) = P$, and subjective probabilities are reduced to the objective ones.

The value function is given by

$$v(x) = \begin{cases} x^\gamma & \text{if} \quad x \geq 0 \\ -\lambda(-x)^\gamma & \text{if} \quad x < 0 \end{cases}, \tag{3}$$

where gains and losses x are derived relative to a specific reference point $\bar{x}$. The function (3) exhibits loss aversion when $\lambda > 1$, and allows for risk aversion in gains but risk seeking in losses when $\gamma < 1$. Figure 1 illustrates the value function for a few parameter combinations, and with a zero reference return. You see that loss aversion causes the value function to be kinked at the reference point. Also, as γ decreases, the value function becomes more concave in gains and convex in losses. Tversky and Kahneman (1992) estimate the value function parameters to $\hat{\lambda} = 2.25$ and $\hat{\gamma} = 0.88$ through individual experiments.[5] These will be used as benchmark parameter estimates in the consequent analysis.

2.1 Incorporating a distributional assumption for X

While Tversky and Kahneman (1992) consider gambles with discrete outcome/probability pairs, this paper considers a random outcome that is continuously distributed. Hence, the expression (1) is derived as:

$$\begin{aligned} U(G;\theta) &= -\int_0^\infty v(x)dw(1-F(x)) + \int_{-\infty}^0 v(x)dw(F(x)) \\ &= \int_0^\infty v(x)w'(1-F(x))f(x)dx + \int_{-\infty}^0 v(x)w'(F(x))f(x)dx \end{aligned} \tag{4}$$

where $F(\cdot)$ is the cumulative distribution function, $f(\cdot)$ is the corresponding probability density function, $v(\cdot)$ is given by (3), $w(\cdot)$ is given by (2), and θ is a vector of utility-depending parameters.[6] The expression (4) is defined over two separate integrals, since the probability weighting of (2) applies differently to gains and losses.

[5] A loss aversion of 2.25 implies that individual investors find the disutility of a loss to be 2.25 times greater than the utility of an equivalent gain. Hence, the loss-averse investor is unwilling to take on a fifty-fifty chance of winning \$200 or losing \$100.

[6] This way of deriving prospective utility is also considered by, e.g., Barberis and Huang (2005).

3 The normal inverse Gaussian distribution

The NIG distribution is presented by Barndorff-Nielsen (1997) in an application to stochastic volatility modeling. It is a mixture of the normal distribution and the inverse Gaussian (IG) distribution. Formally, if a normally distributed variable X has its variance drawn from the IG distribution, i.e.,

$$X|Z = z \sim N(\mu, z),$$

where

$$Z \sim IG(\delta, \sqrt{\alpha^2 - \beta^2}),$$

then X is NIG distributed with parameters α, β, μ, and δ.[7] In this paper, I apply a result of Eriksson, Forsberg, and Ghysels (2005), and therefore adopt the standard parametrization they use, where $\bar{\alpha} = \delta\alpha$ and $\bar{\beta} = \delta\beta$. The $NIG(\bar{\alpha}, \bar{\beta}, \mu, \delta)$ density function is given by

$$f_{NIG}(x; \bar{\alpha}, \bar{\beta}, \mu, \delta) = \frac{\bar{\alpha}}{\pi\delta} \exp\left(\sqrt{\bar{\alpha}^2 - \bar{\beta}^2} - \frac{\bar{\beta}\mu}{\delta}\right) \frac{K_1\left[\bar{\alpha}\sqrt{1 + (\frac{x-\mu}{\delta})^2}\right]}{\sqrt{1 + (\frac{x-\mu}{\delta})^2}} \exp\left(\frac{\bar{\beta}}{\delta}x\right), \tag{5}$$

where K_1 is the modified Bessel function of third order with index 1. The mean, variance, skewness, and kurtosis of $X \sim NIG(\bar{\alpha}, \bar{\beta}, \mu, \delta)$ are given by

$$E[X] = \mu + \frac{\bar{\beta}\delta}{\sqrt{\bar{\alpha}^2 - \bar{\beta}^2}}, \tag{6}$$

$$V[X] = \frac{\delta^2\bar{\alpha}^2}{(\bar{\alpha}^2 - \bar{\beta}^2)^{3/2}}, \tag{7}$$

$$S[X] = \frac{3\bar{\beta}}{\bar{\alpha}(\bar{\alpha}^2 - \bar{\beta}^2)^{1/4}}, \tag{8}$$

and

$$K[X] = \frac{12\bar{\beta}^2 + 3\bar{\alpha}^2}{\bar{\alpha}^2\sqrt{\bar{\alpha}^2 - \bar{\beta}^2}}. \tag{9}$$

While the normal distribution has zero skewness and kurtosis equal to three, equations (8) and (9) show that a NIG distributed variable has parameter-dependent skewness and kurtosis. Explicitly, the parameters of the NIG

[7] To read more on the NIG distribution and its use in stochastic volatility modeling, see Andersson (2001) and Forsberg (2002).

density can be interpreted as follows: $\bar{\alpha}$ and $\bar{\beta}$ are shape parameters with $\bar{\beta}$ expressing the skewness of the distribution, and, when $\bar{\beta} = 0$, $\bar{\alpha}$ expressing the amount of excess kurtosis. The parameter μ is a location parameter and δ is a scale parameter.

3.1 Example

To illustrate the NIG distribution's ability to capture the characteristics of financial returns distributions, consider the monthly returns on the S&P 500 composite index provided by Ibbotson Associates. Table 1 reports on summary statistics. The data is statistically shown to be non-normal by way of a highly significant Jarque-Bera statistic. Figure 2 illustrates both the empirical distribution for the stock returns (panel A) as well as two approximated distributions, where both NIG and normality are assumed (panel B). Clearly, the NIG distribution captures the characteristics of the empirical distribution (positively skewed and leptokurtic) better than the normal distribution does.

3.2 An alternative parameterization involving cumulants

One objective of this paper is to study the relationship between the mean, variance, skewness and kurtosis of a portfolio returns distribution and CPT utility. This is possible by assuming a normal inverse Gaussian distribution for returns. However, it would be easier to establish a connection if we were able to parametrize the NIG distribution as a function of its mean, variance, skewness and kurtosis directly, instead of indirectly via the distribution parameters. Such a parameterization would imply that an individual moment's influence on utility could be analyzed in isolation, i.e., without affecting other moments. Eriksson et al. (2005) show that if the first four *cumulants* of X exist (and fulfill a regularity condition), the NIG density can be expressed as a function of these first four cumulants.[8] This result is a very useful, since the first and second cumulants equal mean and variance, respectively, and skewness and kurtosis are simple normalizations of the third and forth cumulants, as is defined below.

[8] Cumulants are a set of descriptive constants of a distribution, just like moments are. Let μ denote the mean of X, and the second to forth central moments be given by $\mu_i = E[(X-\mu)^i]$, $i = 2, 3, 4$, where $E[\cdot]$ is the expectations operator. The first cumulant, denoted κ_1, equals μ. The second to third cumulants are given by: $\kappa_i = \mu_i$, $i = 2, 3$, and $\kappa_4 = \mu_4 - 3\mu_2^2$. See chapter 3 of Kendall and Stuart (1963) for more on moments and cumulants.

Let κ_1, κ_2, κ_3, and κ_4 denote the first four cumulants of the probability distribution of a stochastic variable X. The mean, variance, skewness, and kurtosis of X are given by

$$E[X] = \kappa_1, \tag{10a}$$
$$V[X] = \kappa_2, \tag{10b}$$
$$S[X] = \frac{\kappa_3}{\kappa_2^{3/2}}, \tag{10c}$$
$$K[X] = \frac{\kappa_4}{\kappa_2^2} + 3. \tag{10d}$$

Eriksson et al. (2005) show that, using (6)-(9) and (10a)-(10d), the NIG parameters $\bar{\alpha}$, $\bar{\beta}$, μ, and δ can be expressed as functions of the first four cumulants κ_1, κ_2, κ_3, and κ_4. The following parameter transformations are presented:

$$\bar{\alpha} = 3\frac{4/\rho + 1}{\sqrt{1-\rho^{-1}}}\frac{\kappa_2^2}{\kappa_4}, \tag{11a}$$
$$\bar{\beta} = 3\frac{\text{signum}(\kappa_3)}{\sqrt{\rho}}\frac{4/\rho + 1}{\sqrt{1-\rho^{-1}}}\frac{\kappa_2^2}{\kappa_4}, \tag{11b}$$
$$\mu = \kappa_1 - \frac{\text{signum}(\kappa_3)}{\sqrt{\rho}}\sqrt{(12/\rho + 3)\frac{\kappa_2^3}{\kappa_4}}, \tag{11c}$$
$$\delta = \sqrt{3(4/\rho + 1)(1-\rho^{-1})\frac{\kappa_2^3}{\kappa_4}}, \tag{11d}$$

where $\rho = 3\kappa_4\kappa_2\kappa_3^{-2} - 4$.[9] The transformation is valid under the regularity condition: $\rho > 1$.[10]

Equations (5) and (11a)-(11d) imply that there exists an alternative parametrization of the NIG density, denoted $\bar{f}_{NIG}$, that depends directly on the first four cumulants, i.e., $\bar{f}_{NIG} = \bar{f}_{NIG}(x; \{\kappa_i\}_{i=1}^4)$. Using this alternative NIG density, one can approximate an empirical distribution by estimating its first four cumulants $\{\kappa_i\}_{i=1}^4$, instead of estimating the standard NIG distribution parameters $\bar{\alpha}$, $\bar{\beta}$, μ and δ. As will be shown in the next section, this transformation is most useful for the purpose of this paper.

[9] The function signum(x) is congruent with the sign of x.

[10] This condition implies that the distribution is "well-behaved", i.e., NIG approximations do not exist for all cumulant values.

4 Analyzing prospective utility and central moments

4.1 Investor utility under NIG

Consider a single-period investor who values a risky portfolio according to CPT. Further, assume that the portfolio outcome (return) is NIG distributed. Unfortunately, this assumption has a drawback, namely, the NIG cumulative distribution function does not exist in closed form. This implies that probability weighting cannot be considered, since, as (4) shows, the weighting function $w(\cdot)$ has the cumulative distribution function as an argument. Hence, I assume that $\tau = 1$ in (2), reducing the subjective probabilities to the objective ones, i.e., $w(P) = P$. The plausible implications of this assumption will be discussed in respect to the results obtained.

Under the assumptions of NIG distributed returns and objective probabilities, the expression $E_w[v(X)]$ of (1) reduces to $E[v(X)]$, and (4) simplifies to

$$U(G;\theta) \equiv \tilde{U}(\theta) = \int_0^{\infty} x^{\gamma} \bar{f}_{NIG}(x;\{\kappa_i\}_{i=1}^4)dx - \lambda \int_{-\infty}^{0} (-x)^{\gamma} \bar{f}_{NIG}(x;\{\kappa_i\}_{i=1}^4)dx, \tag{12}$$

where the alternative NIG density parameterization $\bar{f}_{NIG}$ is employed. Expression (12) drives the *expected* prospect theory utility, which I, henceforth, refer to as prospective utility. Utility parameters are gathered in $\theta = (\gamma, \lambda, \bar{x}, \{\kappa_i\}_{i=1}^4)'$, where γ reflects risk aversion in gains and risk-seeking in losses, λ measures loss aversion, $\bar{x}$ is the reference return, and $\{\kappa_i\}_{i=1}^4$ are the first four cumulants of the outcome distribution. Although the reference return $\bar{x}$ does not appear in (12), it is included in θ since the outcomes x are derived as deviations from $\bar{x}$.

What is striking about prospective utility, specified by (12), is that the first four cumulants enter as function parameters. Recall that cumulants are closely related to central moments, as (10a)-(10d) shows. Using (12), you can in fact study prospective utility as a function of, for instance, skewness simply by varying κ_3, or as a function of kurtosis by varying κ_4. Such an analysis is conducted in the next subsection.

4.2 Utility as a function of central moments

How does prospective utility respond to changes in the gamble return mean, variance, skewness and kurtosis? To answer this question, I study $\tilde{U}(\theta)$ as a function of, e.g., skewness, by calibrating all parameters of $\tilde{U}(\theta)$ to reasonable estimates, and by then varying κ_3, which is proportional to skewness.[11] An analysis of mean, variance, and kurtosis is done in similar fashion. In calibrating γ and λ, I choose the estimates of Tversky and Kahneman (1992), i.e., $\hat{\gamma} = 0.88$ and $\hat{\lambda} = 2.25$. The cumulants $\{\kappa_i\}_{i=1}^4$ are estimated using historical monthly data on S&P 500 composite index returns. The sample period is January 1926 to December 2003, yielding 936 observations. Furthermore, the investor reference return $\bar{x}$ is set to the risk-free interest rate, measured by the average return on a U.S. 30-day Treasury bill. Summary statistics are found in table 1.

4.2.1 Mean and variance

Figure 3 (panel A) plots prospective utility (12) as a function of mean, together with a plot of the two perimeter distributions (panel B), i.e., the distributions with smallest and largest analyzed mean. The reference point $\bar{x}$, which separates gains from losses, is also indicated in panel B. When the mean increases, prospective utility follows. The intuition is that a higher mean reduces the probability for a loss (relative to $\bar{x}$), increasing utility.

In figure 4 (panel A), prospective utility is plotted as a function of variance. The distributions with smallest and largest variance are plotted in panel B of figure 4, along with a mark for the reference return. Prospective utility decreases when variance increases, which is intuitively understandable since a higher variance spreads the distribution around its mean, increasing the probability for a loss.

The results of figures 3 and 4 imply that CPT investors choose mean-variance efficient portfolios, i.e., portfolios with minimum variance for a given mean. This result supports the analytical findings of, e.g., Barberis and Huang (2005), who show that CPT investors indeed choose mean-variance efficient portfolios, however, under a normality assumption for returns. The results presented in this paper assume that gamble outcomes are NIG distributed, which is a more general assumption.

4.2.2 Skewness and kurtosis

To obtain an apparent view of prospective utility's dependence on skewness and kurtosis, figures 5 and 6 show illustrative graphs of these relations. Panel A of figure 5 plots $\tilde{U}$ for a skewness between -2 and 2. A slightly hump-shaped curve is shown, where the slope is positive at very low skewness and turns negative as skewness increases. For reasonable estimate of skewness, i.e., in

[11] For a specific set of parameter estimates $\hat{\theta}$, $\tilde{U}(\hat{\theta})$ is derived using numerical quadrature in Matlab programming language. The function *quad* is applied.

between -1 and 1, utility falls as skewness rises. Panel B of figure 5 plots the two perimeter distributions where skewness either equals -2 or 2, along with a mark to point out the reference return. When skewness increases, the tail mass shifts from negative to positive returns, which has a positive effect on utility. However, the center mass moves in the opposite direction to preserve the mean, which makes moderate-sized losses more probable. Hence, a (mean-preserving) rise in skewness causes for two counter-working effects on utility. As the distribution becomes so skewed that its top crosses to the left side of the reference point, the effect on utility is on balance negative, due to loss aversion.

Figure 6 (panel A) presents prospective utility plotted against a kurtosis between 3 and 20. The graph is clearly positively sloped, meaning that prospective utility increases with kurtosis. Panel B of figure 6 helps to establish an intuitive explanation for this result. When kurtosis increases, both tail masses increase while the center mass becomes more concentrated around the mean. Although extreme negative returns become more likely, moderate-sized ones become less likely. This second effect overshadows the first, causing prospective utility to rise.

4.3 Comparative statics over γ and λ

The previous analysis assumed Tversky and Kahneman (1992) estimates for the value function (3) parameters γ and λ. How are the results affect by changes in these parameters? One way to analyze this is by deriving derivatives of $\tilde{U}(\theta)$ with respect to mean (E), variance (V), skewness (S), and kurtosis (K), for different value-combinations of γ and λ. The derivatives are obtained knowing that

$$\frac{\partial \tilde{U}(\theta)}{\partial E} = \frac{\partial \tilde{U}(\theta)}{\partial \kappa_1}, \tag{13}$$

$$\frac{\partial \tilde{U}(\theta)}{\partial V} = \frac{\partial \tilde{U}(\theta)}{\partial \kappa_2}, \tag{14}$$

$$\frac{\partial \tilde{U}(\theta)}{\partial S} = \frac{\partial \tilde{U}(\theta)}{\partial \kappa_3}\frac{\partial \kappa_3}{\partial S} + \frac{\partial \tilde{U}(\theta)}{\partial \kappa_2}\frac{\partial \kappa_2}{\partial S}, \tag{15}$$

$$\frac{\partial \tilde{U}(\theta)}{\partial K} = \frac{\partial \tilde{U}(\theta)}{\partial \kappa_4}\frac{\partial \kappa_4}{\partial K} + \frac{\partial \tilde{U}(\theta)}{\partial \kappa_2}\frac{\partial \kappa_2}{\partial K}, \tag{16}$$

where the last two derivatives use (10c) and (10d), respectively, together with the chain rule.

It would be ideal if the derivatives of (13)-(16) could be expressed in closed form. Unfortunately, this is to my knowledge impossible since $\tilde{U}(\theta)$ is too complex a creature. Instead, I apply Leibnitz' rule and first differentiate the integrands of (12) analytically, followed by numerical quadrature to approximate the integrations.[12] Such a numerical technique requires estimates of the first four cumulants of an arbitrary portfolio returns distribution. Therefore, I again calibrate $\{\kappa_i\}_{i=1}^{4}$ to empirical estimates from the S&P 500 composite index return data previously used.

Table 2 presents numerical derivatives of prospective utility with respect to the mean, variance, skewness and kurtosis of the gamble outcome distribution.[13] Consider the case when $\gamma = 1$, i.e., the value function (3) is (piecewise) linear. Risk-neutrality ($\lambda = 1$) implies a one-to-one relationship between utility and mean ($\frac{\partial \tilde{U}}{\partial E} = 1$), and that the investor is indifferent to a mean-preserving spread ($\frac{\partial \tilde{U}}{\partial V} = 0$). When $\lambda > 1$, you see that $\frac{\partial \tilde{U}}{\partial E}$ is positive, irrespective of values for γ and λ. Also, $\frac{\partial \tilde{U}}{\partial V}$ is negative, irrespective of values for γ and λ. Hence, prospective utility increases (decreases) with the mean (variance). These effects are even more pronounced when the investor is extensively risk averse ($\lambda = 3$), which is intuitive. The results, again, shows that CPT investors choose mean-variance efficient portfolios.

When $\gamma < 1$, prospective utility is less sensitive to changes in mean and variance. This is natural since the value function is less sensitive to differences in returns, due to the decreasing (increasing) marginal utility for gains (losses) as figure 1 shows.

For skewness and kurtosis, no clear pattern can be found, although it seems like prospective utility decreases when skewness rises and increases with kurtosis, which is the case when $\gamma < 1$ and $\lambda > 1$.

4.4 Differences compared to power utility?

Are the previously obtained results unique for prospect theory investors? To analyze this, figures 3-6 present the dependence of standard concave utility to characteristics of financial returns, in addition to prospective utility. Specifically, I choose power utility as a comparison because of its wide use in the portfolio choice literature, and because power utility investors care about higher order moments, as opposed to, e.g., mean-variance investors.

Single-period expected power utility under NIG distributed returns is derived according to (4) with $v(x)$ replaced by $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$, where $w = 1 + \frac{x}{100}$, since power utility investors derive utility from final wealth states.

[12] Leibnitz' rule applies, since the integrand is a continuous function of the cumulants.

[13] I should add that, when calculating these derivatives, $\frac{\partial \kappa_2}{\partial S}$ and $\frac{\partial \kappa_2}{\partial K}$ are assumed to equal zero, since κ_2 is only a normalization factor for skewness and kurtosis. A marginal change in skewness and kurtosis is, hence, only induced by changes in κ_3 and κ_4, respectively.

The parameter of relative risk aversion γ is assumed to equal 3. Generally, I find that power utility shows minor responses to moment changes. The utility function displays local risk-neutrality. However, some evidence of mean-variance efficiency is shown (as we would expect) in figures 3 and 4, i.e., utility rises in mean and falls in variance.

Contrary to prospective utility, figure 5 displays slight evidence that power utility increases with skewness. This does not come as a surprise since power utility fulfills the condition of non-increasing absolute risk aversion, which implies a preference for skewness (Arditti, 1967). Figure 6 shows a slight negative relation between kurtosis and power utility contrary to the result for prospective utility.

Hence, I find that CPT preferences for higher moments are unique, at least when compared with power utility. The driving source of this difference is that CPT investors value gains and losses asymmetrically, displaying loss aversion, while power utility investors are locally risk-neutral.

4.5 Robustness to choice of data

The analysis in this section has used empirical data to represent the portfolio returns distribution. The alternative NIG density's cumulant parameters have been calibrated using "reasonable" estimates. Are the obtained results sensitive to the choice of portfolio outcome distribution? Generally, I find that this is not the case. I have tried using other stock market indices as well as bond market data, and the results have shown to be robust to such modifications. Also, a change in data-horizon has been considered without any dramatic effects on the obtained results.

5 Portfolio choice with NIG distributed returns

In this section, I analyze the single-period portfolio choice of CPT investors over stocks, bonds, and a risk-free asset. Such an analysis is not new. For instance, Aït-Sahalia and Brandt (2001) present the unconditional portfolio choice of investors with various different preferences, among them CPT. Nevertheless, I study the portfolio choice of CPT investors assuming returns are NIG distributed in comparison with a normality assumption. Since the normal distribution neglects moments higher than two, in contrast to NIG, this allows for an isolated study of how preferences over higher moments affect CPT portfolio choice. The same data set as previously employed is used in this exercise.[14]

The portfolio choice problem is carried out as follows: The investor optimizes weights in a risky portfolio consisting of both stocks (q_s) and bonds (q_b), with the alternative of investing in a bills portfolio ($1 - q_s - q_b$), having risk-free return equal to the mean Treasury bill return. Prospective utility is derived with a reference point equal to this the same risk-free return. Hence, the bills portfolio has zero prospective utility. Therefore, the investor has the following objective:

$$\max_{q_s,q_b} \tilde{U}(\theta) = \int_0^{\infty} x^{\gamma} f(x;\xi)dx - \lambda \int_{-\infty}^{0} (-x)^{\gamma} f(x;\xi)dx,$$

subject to

$$x = q_s x_s + q_b x_b \,,$$

and

$$\begin{aligned} q_s, q_b &\in [0,1], \\ q_s + q_b &\leq 1, \end{aligned}$$

where x_s (x_b) is the stochastic return on stocks (bonds), $f(x;\xi)$ is an arbitrary probability density function, and $\theta = (\gamma, \lambda, \xi)'$ is a parameter vector consisting of all utility-depending parameters. The weight on Treasury bills is $q_{tb} = 1 - q_s - q_b$. The portfolio choice is carried out over one-, six-, and twelve-month horizons.[15]

Tables 3 and 4 report on optimal portfolio weights for CPT investors under the assumption that x is NIG and normally distributed, respectively.

[14] See table 1 for summary statistics.

[15] The Matlab constrained minimization routine *fmincon* is applied to solve the portfolio choice problem.

First, observe the large horizon effects, i.e., larger weights are placed on stocks as the horizon increases (irrespective of distributional assumption). This shows that stocks become more attractive over longer horizons, and, also, confirms the findings of Benartzi and Thaler (1995) that loss-averse investors are myopic.[16]

Second, there is a considerable difference in the weight on stocks under the NIG assumption compared to normality. An overall larger weight is placed on stocks under NIG, which reflects that CPT investors find stocks more attractive when higher order moments are taken into consideration. Studying table 1, you notice that, although the stock market returns distribution is only mildly skewed, its kurtosis is extensively larger than three, which is assumed by normality. This is the case irrespective of the returns horizon. Following the results in the previous section, kurtosis has a positive effect on prospective utility. The reason why CPT investors place a larger weight on stocks under a NIG compared to a normality assumption is most likely related to the large observed kurtosis of the empirical stock returns distribution. Although a high kurtosis induces a larger probability for extreme losses, moderate-sized losses become less probable, since the distribution becomes more centered around its mean. Under the assumption of a NIG distribution, this effect is accounted for, implying that CPT investors put more on stocks than they would under normality.

[16] Aït-Sahalia and Brandt (2001) obtain a similar result.

6 Conclusions and ending discussion

This paper studies prospective utility under the assumption of a NIG distributed single-period gamble outcome. Such an assumption takes the higher moments of financial returns distributions into account, as opposed to the normality assumption, where only mean and variance are considered. Using numerical techniques, and a model calibration to empirical parameter estimates, I find that: (a) CPT investors choose mean-variance efficient portfolios, (b) prospective utility generally falls when skewness increases, (c) prospective utility increases with kurtosis, (d) the CPT investor's portfolio choice displays large horizon effects, i.e., a larger weight is placed on stocks if the horizon is longer, and (e) when a more general distributional assumption is made on returns, i.e., the NIG distribution, CPT investors place a larger weight on stocks, relative to a normality assumption.

The main contributions of this paper are (b), (c), and (e). What do these results tell us? Having a choice between two single-period gambles with stochastic outcomes, which one does the CPT investor choose? Judging from the central moments of the outcome distribution, the results of this paper tells that, all other things equal, the investor chooses the one gamble with higher kurtosis and/or lower skewness. This result is surprising since you often believe skewness is a good thing for prospective utility, since extreme losses are less probable. However, what you then forget is that a (positively) skewed distribution has a larger mass closer to, or even to the left side of, the investors reference return. This large probability for a small or moderate-sized loss is extensively detrimental for prospective utility.

Similarly, you would often believe that kurtosis is a bad thing, since it involves a large probability for extreme losses. However, you forget that a high kurtosis concentrates the distribution around its mean, making moderate-sized losses considerably less probable. This effect has a significantly positive impact on prospective utility.

The results of this paper have implications for portfolio choice. I show that, when prospect theory investors take higher order moments into account, they find stocks more attractive than they do when considering only mean and variance. This result is likely related to the high kurtosis of the stock returns distribution, an attractive feature for the prospect theory investor.

In this paper, the assumption of NIG distributed portfolio returns has limited the analysis to disregard subjective probability weighting. Would the inclusion of this ingredient affect the obtained results? This question is not easily answered. However, it could be the case that the answer is yes. Since the weighting function (2) overweights small probabilities, the balanced outcome of the two counter-working effects on prospective utility, following an increase in skewness or kurtosis, might be different. Then again, loss aversion has such a large impact in this paper's application of prospect theory that the inclusion of probability weighting would, I believe, only have minor implications.

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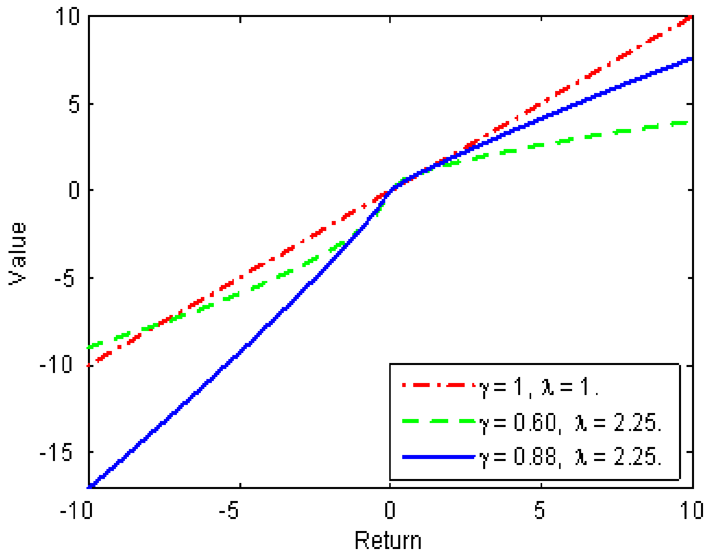
Appendix

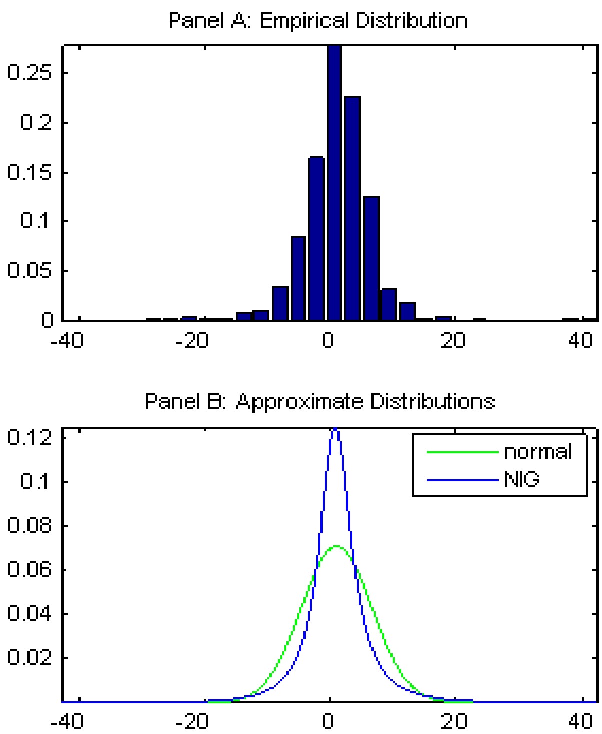
A Figures and tables

Figure 1: The Value Function



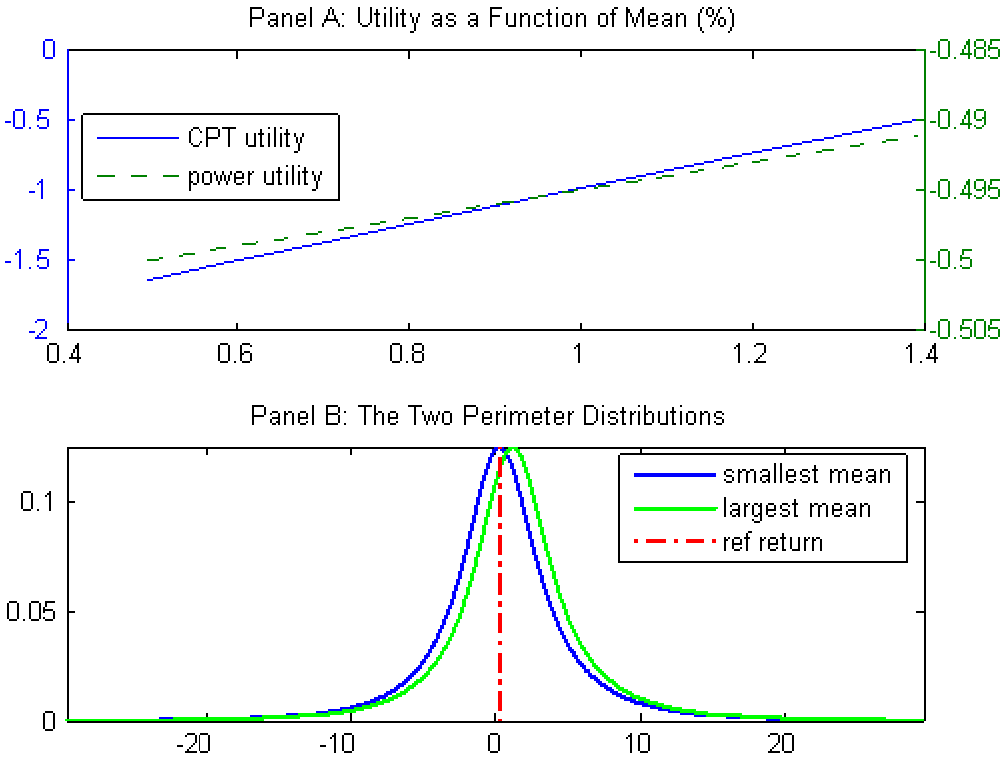
This figure illustrates the CPT value function over returns (%) with a zero reference point. When $\gamma = \lambda = 1$, the investor is risk-neutral and has a linear value function. Loss aversion enters when $\lambda > 1$, causing a dramatic jump in marginal utility at the reference point. As γ increases the function becomes more S-shaped. Tversky and Kahneman (1992) estimate the parameters to $\hat{\gamma} = 0.88$ and $\hat{\lambda} = 2.25$.

Figure 2: Empirical and Approximate Distributions



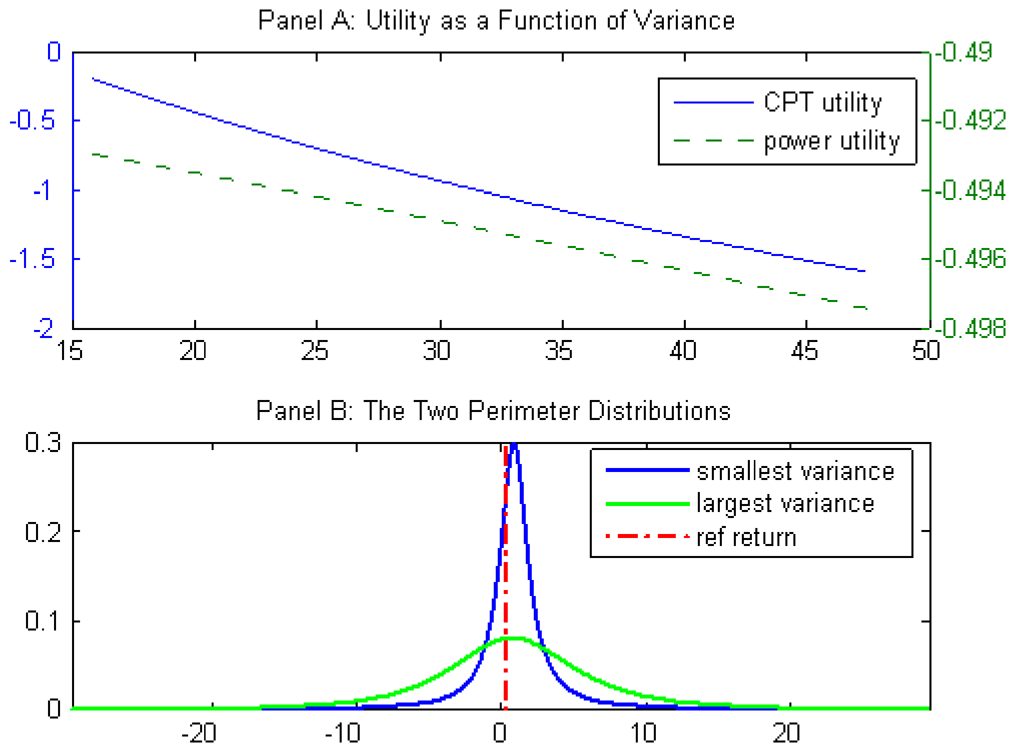
This figure exemplifies how well the NIG distribution approximates an empirical distribution of financial returns. Panel A shows the actual empirical distribution of monthly S&P 500 returns (%) from 1926:1 to 2003:12. Panel B presents its NIG approximation along with the normal approximation as a comparison.

Figure 3: Utility in Relation to Mean



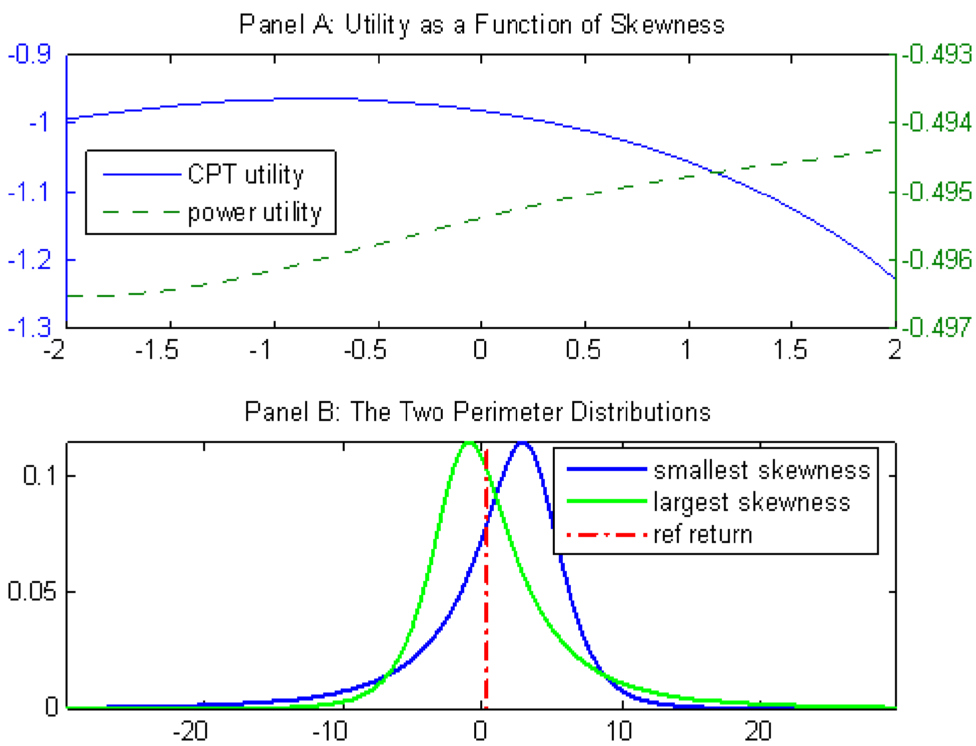
This figure plots CPT and power utility as a function of mean (panel A), and the two perimeter distributions (panel B), i.e., with smallest and largest interval-mean, respectively. CPT parameter estimates for γ and λ follow Tversky and Kahneman (1992). The power utility parameter of constant relative risk aversion is set to 3. In Panel A, CPT (power) utility is measured on the left (right) axis.

Figure 4: Utility in Relation to Variance



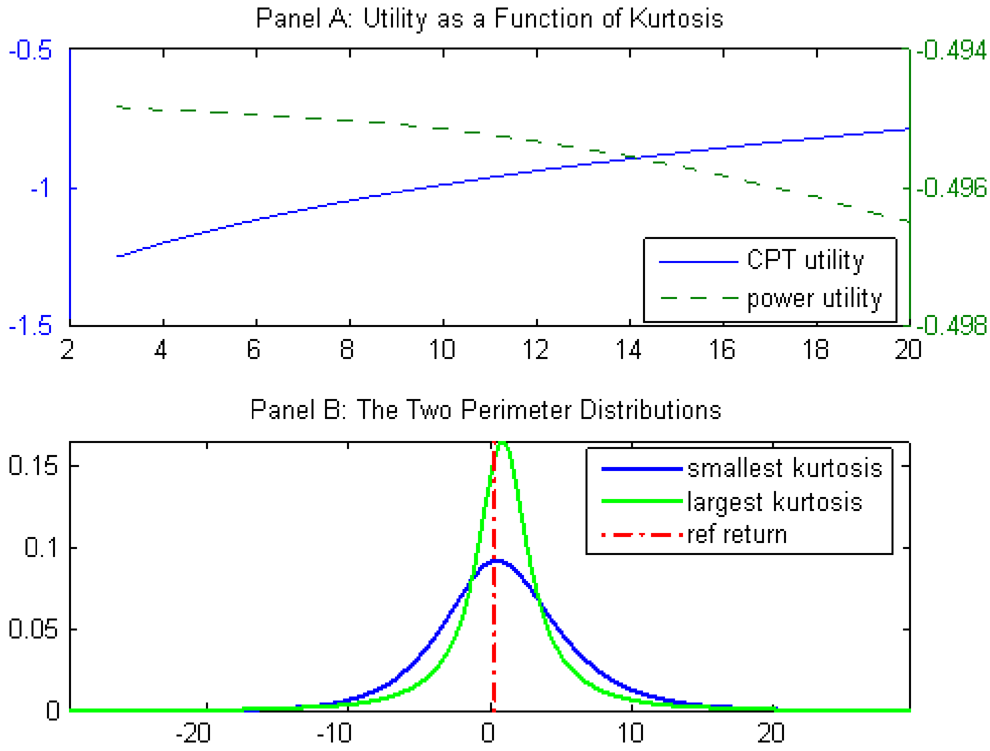
This figure plots CPT and power utility as a function of variance (panel A), and the two perimeter distributions (panel B), i.e., with smallest and largest interval-variance, respectively. CPT parameter estimates for γ and λ follow Tversky and Kahneman (1992). The power utility parameter of constant relative risk aversion is set to 3. In Panel A, CPT (power) utility is measured on the left (right) axis.

Figure 5: Utility in Relation to Skewness



This figure plots CPT and power utility as a function of skewness (panel A), and the two perimeter distributions (panel B), i.e., where skewness equals either -2 or 2 *ceteris paribus*. CPT parameter estimates for γ and λ follow Tversky and Kahneman (1992). The power utility parameter of constant relative risk aversion is set to 3. In Panel A, CPT (power) utility is measured on the left (right) axis.

Figure 6: Utility in Relation to Kurtosis



This figure plots CPT and power utility as a function of kurtosis (panel A), and the two perimeter distributions (panel B), i.e., where kurtosis equals either 3 or 20 *ceteris paribus*. CPT parameter estimates for γ and λ follow Tversky and Kahneman (1992). The power utility parameter of constant relative risk aversion is set to 3. In Panel A, CPT (power) utility is measured on the left (right) axis.

Table 1: Summary Statistics for Financial Returns, Jan. 1926 - Dec. 2003

	S&P 500			U.S. long-term bond			U.S. 30-day bill		
	Horizon (months)			Horizon (months)			Horizon (months)		
	1	6	12	1	6	12	1	6	12
Mean (%)	0.986	6.086	12.614	0.465	2.815	5.779	0.308	1.873	3.816
Max (%)	42.564	100.216	162.892	15.235	35.701	54.405	1.348	7.708	15.201
Min (%)	-29.726	-51.159	-67.559	-9.822	-18.798	-17.091	-0.062	-0.070	-0.044
Std. dev. (%)	5.623	14.295	22.082	2.267	5.837	9.010	0.256	1.542	3.133
Skewness	0.391	0.162	0.350	0.682	0.923	1.452	0.989	0.928	0.909
Kurtosis	12.453	6.406	6.466	8.103	6.202	6.900	4.121	3.794	3.672
Jarque-Bera	3508.6	441.29	478.18	1088.2	535.44	905.56	201.6	149.45	144.06
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
No. obs.	936	931	925	936	931	925	936	931	925

This table reports on summary statistics for returns on the S&P 500 composite index, a long-term U.S. government bond, and a U.S. 30-day Treasury bill. The time period stretches from January 1926 to December 2003. One-month, six-month, and twelve-month horizons are considered. An overlapping window is used to derive the lower frequency returns. Jarque-Bera is a test over skewness and kurtosis under the null of normality, where skewness equals zero and kurtosis is equal to three. Note that p-values are in parentheses. Source: Ibbotson Associates.

Table 2: Numerical Derivatives of Prospective Utility

$\frac{\partial \tilde{U}}{\partial E}$		λ			$\frac{\partial \tilde{U}}{\partial V}$		λ		
		1	2.25	3			1	2.25	3
	0.6	0.5288	0.8256	1.0037		0.6	-0.0042	-0.0258	-0.0388
γ	0.88	0.8178	1.2635	1.5309	γ	0.88	-0.0019	-0.0430	-0.0676
	1	1.0000	1.5394	1.8630		1	0.0000	-0.0537	-0.0860
$\frac{\partial \tilde{U}}{\partial S}$		λ			$\frac{\partial \tilde{U}}{\partial K}$		λ		
		1	2.25	3			1	2.25	3
	0.6	-0.0905	-0.1399	-0.1700		0.6	-0.0206	0.0086	0.0162
γ	0.88	-0.0472	-0.0685	-0.0813	γ	0.88	-0.0162	0.0103	0.0263
	1	-0.0011	0.0107	0.0171		1	-0.0205	0.0111	0.0301

This table presents numerical derivatives of single-period prospective utility, given by

$$\tilde{U}(\theta) = \int_0^{\infty} x^{\gamma} \bar{f}_{NIG}(x; \{\kappa_i\}_{i=1}^4) dx - \lambda \int_{-\infty}^{0} (-x)^{\gamma} \bar{f}_{NIG}(x; \{\kappa_i\}_{i=1}^4) dx,$$

with respect to the underlying portfolio distribution mean (M), variance (V), skewness (S), and kurtosis (K). See equations (13)-(16). The empirical distribution of monthly returns of the S&P 500 composite index approximate the portfolio distribution, i.e., provides estimates for $\{\kappa_i\}_{i=1}^4$. The average return on a U.S. 30-day Treasury bill represents the investor's reference return, distinguishing gains from losses. Prospect theory parameters λ and γ are varied; as λ increases, the investor becomes more loss averse; as γ increases, the investor becomes more risk-averse in gains but more risk-seeking in losses.

Table 3: Single-Period Portfolio Choice of Cumulative Prospect Theory Investors Under a NIG Assumption

		One-Month Horizon			Six-Month Horizon			Twelve-Month Horizon		
		q_s	q_b	q_{tb}	q_s	q_b	q_{tb}	q_s	q_b	q_{tb}
	$\gamma = 0.6$	1	0	0	1	0	0	1	0	0
$\lambda = 1$	$\gamma = 0.88$	1	0	0	1	0	0	1	0	0
	$\gamma = 1$	1	0	0	1	0	0	1	0	0
	$\gamma = 0.6$	0.40	0.60	0	0.60	0.40	0	0.89	0.11	0
$\lambda = 2.25$	$\gamma = 0.88$	0.16	0.37	0.47	0.58	0.42	0	1	0	0
	$\gamma = 1$	0.07	0	0.93	0.57	0.43	0	1	0	0
	$\gamma = 0.6$	0.21	0.34	0.45	0.46	0.54	0	0.61	0.39	0
$\lambda = 3$	$\gamma = 0.88$	0.07	0.16	0.76	0.37	0.63	0	0.59	0.41	0
	$\gamma = 1$	0.06	0.09	0.85	0.35	0.65	0	0.60	0.40	0

This table shows optimal portfolio weights in stocks (q_s), bonds (q_b), and Treasury bills (q_{tb}), for single-period CPT investors with value function:

$$v(x) = \begin{cases} x^\gamma & \text{if} \quad x \geq 0 \\ -\lambda(-x)^\gamma & \text{if} \quad x < 0 \end{cases},$$

where returns x are derived relative to a reference return $\bar{x}$ equal to the average return on Treasury bills. Returns x are assumed normal inverse Gaussian distributed. The investor horizon is either one, six, or twelve months. Summary statistics for these data are found in table 1. Values for parameters γ and λ are varied. Restrictions $q_s, q_b \in [0, 1]$ and $q_s + q_b \leq 1$ are imposed in the optimization.

Table 4: Single-Period Portfolio Choice of Cumulative Prospect Theory Investors Under a Normality Assumption

		One-Month Horizon			Six-Month Horizon			Twelve-Month Horizon		
		q_s	q_b	q_{tb}	q_s	q_b	q_{tb}	q_s	q_b	q_{tb}
	$\gamma = 0.6$	1	0	0	1	0	0	1	0	0
$\lambda = 1$	$\gamma = 0.88$	1	0	0	1	0	0	1	0	0
	$\gamma = 1$	1	0	0	1	0	0	1	0	0
	$\gamma = 0.6$	0.12	0.55	0.32	0.42	0.58	0	0.57	0.43	0
$\lambda = 2.25$	$\gamma = 0.88$	0.12	0.45	0.43	0.45	0.55	0	0.83	0.17	0
	$\gamma = 1$	0	0.57	0.43	0.47	0.53	0	1	0	0
	$\gamma = 0.6$	0	0.46	0.54	0.34	0.65	0	0.43	0.57	0
$\lambda = 3$	$\gamma = 0.88$	0	0.50	0.50	0.34	0.66	0	0.45	0.55	0
	$\gamma = 1$	0	0.63	0.37	0.34	0.66	0	0.47	0.53	0

This table shows optimal portfolio weights in stocks (q_s), bonds (q_b), and Treasury bills (q_{tb}), for single-period CPT investors with value function:

$$v(x) = \begin{cases} x^\gamma & \text{if} \quad x \geq 0 \\ -\lambda(-x)^\gamma & \text{if} \quad x < 0 \end{cases},$$

where returns x are derived relative to a reference return $\bar{x}$ equal to the average return on Treasury bills. Returns x are assumed normally distributed. The investor horizon is either one, six, or twelve months. Summary statistics for these data are found in table 1. Values for parameters γ and λ are varied. Restrictions $q_s, q_b \in [0, 1]$ and $q_s + q_b \leq 1$ are imposed in the optimization.