25 On the likelihood and relevance of reswitching and reverse capital deepening

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Premise and summary

In this chapter I try to contribute to the debate on the probability of reswitching and on the relevance of this issue. I offer it as a homage to a dear friend, for whose qualities both human and scientific I feel great admiration.

At the cost of some simplification, one can distinguish two main replies to the claim, advanced most forcefully by Garegnani (1970, 1978–79, 1990) but also by many others (including Heinz and myself), that reswitching and reverse capital deepening undermine the entire supply-and-demand (or marginalist, or neoclassical) approach to value, distribution and growth.

The most frequent reply so far has consisted of the argument that neoclassical theory does not need any notion of aggregate capital, as shown by general equilibrium theory in its modern or neo-Walrasian versions (GET for short), and that therefore the neoclassical approach to value and distribution survives reswitching unscathed. However, there are signs that this line of defence is being increasingly recognized as problematical even by economists of neoclassical training, because of growing doubts as to whether modern general equilibrium theory can really support the neoclassical scarcity-based explanations of how distribution and employment are determined in reality. Consciousness seems to be spreading (e.g. Bloise and Reichlin 2009) that applied neoclassical economics continues to rely on its traditional formulations based on the treatment of capital as a single factor of variable 'form', formulations whose implications are not supported by modern general equilibrium theory. The absence of such a support is increasingly admitted, not only because of the well-known problems of GET with uniqueness and stability, but also because of the increasingly recognized difficulty with connecting the results of GET with real economies. Four reasons behind this difficulty may be briefly remembered, roughly in order of decreasing (but growing) general awareness of their relevance. First, modern general equilibria, being determined on the basis of a given *vector* of initial endowments of capital goods (rather than, as traditionally, on the basis of a given 'quantity of capital' of a 'form' endogenously determined by the equilibrium itself, and only slowly altered by accumulation), suffer from *insufficient data persistence*: the data relative to the capital endowments, as well as (in the temporary equilibrium versions) the data relative to expectations, are susceptible of being very quickly altered by disequilibrium actions, so the equilibrium is unable to indicate the situation the economy tends to (the assumption that equilibrium is reached instantaneously so the economy is all the time in equilibrium is patently contradicted by experience).[1] Second, the definition of equilibrium requires either (in the intertemporal versions of GET) an assumption of complete futures markets which has no correspondence with reality (the alternative assumption of perfect foresight is just as unreal and also fraught with logical difficulties), or (in the temporary equilibrium versions) an assumption of given subjective (and therefore unknowable) expectations which, besides creating problems to the definition and to the existence of equilibrium owing to the complications due to diverging expectations, introduces an element of *indeterminateness* that risks depriving the theory of any possibility of definite predictions. Third, these general equilibria suffer from *insufficient factor substitutability*: the very little substitutability among factors once each capital good is treated as a different factor and the 'form' of capital is given implies a high likelihood that many equilibrium factor rentals be zero, and that even the equilibrium real wage may be zero or close to zero, and susceptible of unrealistically drastic changes if labour supply changes slightly: and this deprives the equilibrium of plausibility (Garegnani 1990, pp. 57–8). Fourth, if the traditional foundation of the belief that investment adjusts to full-employment savings – namely, the assumption of a 'well-behaved' substitution between labour and value capital, which is the basis for the thesis of a negative elasticity of aggregate investment with respect to the rate of interest – is admitted to be incompatible with a theory that explicitly wants to do without a value factor 'capital', no other persuasive basis can be found for that belief (Petri 2004, ch. 7); accordingly the full employment of resources becomes a part of the *definition* of general equilibrium, but with no reason to consider it as reflecting he tendential result of the working of market forces.[2]

It is not often realized that the origin of these problems lies precisely in modern general equilibrium theory's attempt to do without the indefensible notion of capital as a single factor of variable 'form'; nonetheless, because of them GET is more and more considered a fascinating intellectual construction but with little relevance for the explanation of how value, distribution and employment are determined in real economies. But then the criticism of traditional neoclassical capital theory can no longer be dismissed, because (as argued in detail in Petri 2004) the traditional neoclassical conception of capital emerges as the true and the sole possible foundation of neoclassical macroeconomics, growth theory, etc. – in short, of the entire neoclassical 'vision'.

The second type of reply has insisted on a supposedly low likelihood of reswitching and of reverse capital deepening. It is more or less explicitly admitted that these phenomena do constitute problems for the neoclassical approach, and it is recognized that these phenomena are definitely possible and that the range of values of technical coefficients for which these phenomena can happen is not of measure zero; but it is argued that they are improbable, implicitly suggesting that one may legitimately assume their absence and the validity of the traditional neoclassical arguments about capital-labour substitution.[3] The analytical attempts to support this thesis have not been many, but Mainwaring and Steedman (2000, p. 346) have spoken of 'cumulating evidence' that the probability of reswitching is 'small'.

The main purpose of the present paper, carried out in the first two sections, is to refute an important part of the 'cumulating evidence' that Mainwaring and Steedman accept: the results (thus far only available in Italian) reached in 1987 by the late Professor D'Ippolito for the Samuelson-Garegnani two-sector model, and accepted also by Schefold (1997b, p. 479) and with great emphasis by Potestio (2010, p. 150). D'Ippolito was the first to try to prove analytically that the *a priori* numerical probability of reswitching and hence of reverse capital deepening is low, by measuring this probability as the ratio between the suitably constructed area or volume of possible coefficient values for which reswitching happens, and the area or volume of all possible coefficient values – essentially the same method then adopted by Laing (1991) and by Mainwaring and Steedman.

The first part of this chapter, after a presentation (that may be useful for economists unable to read Italian) of D'Ippolito's argument, shows that D'Ippolito's very low values for the probability of the so-called 'perverse' switches[4] are due to a logical slip. D'Ippolito obtains probabilities that a switch of techniques be associated with reverse capital deepening which, as the value of the average rate of profits increases from 5% to 30%, increase from approximately 2% to 8%; once the logical slip is corrected, these probabilities rise to, respectively, 36% and 45%! In order to assess this result, the chapter goes on to examine two other ways of estimating these probabilities. A procedure, perhaps closer to the spirit of what D'Ippolito may have had in mind, results in probabilities not so high but still significantly higher than D'Ippolito's: 7.5% and 10.7% respectively for r = 5% and 30% (Table 25.1, last column: 'D'Ippolito reinterpreted'). A quite different approach is then illustrated in the second section, which I think has intuitive appeal because it is based in an immediate way on the shape of the $w(r)$ curves. This method requires the estimation of complicated integrals, but numerical approximation methods make it possible to surmount this difficulty. There result still different and somewhat higher probabilities, e.g. not less than 8.4% and not less than 13.5%, for r = 5% and r = 30% respectively.[5] Table 25.1 reports the probabilities calculated with the different procedures, for values of the rate of profit from 1% to 3000%. The conclusion is that at least for the Samuelson-Garegnani model there is no reason at all to consider reswitching unlikely. The paper by Laing (1991) is argued not to alter this conclusion.

The third and fourth parts are more provisional but I have decided to present them anyway as a stimulus to debate. The third section proposes, for the difference of my results from those of Mainwaring and Steedman (2000), an explanation based on the restrictiveness of their assumption that alternative techniques have in common all goods that influence the shape of the $w(r)$ curve. The fourth part goes on to examine the robustness of the argument that a very low probability of reswitching, if it could be proved, would rehabilitate traditional neoclassical analyses. This requires a discussion of some aspects of investment theory and in particular of the approach to investment of Dornbusch and Fischer.

Table 25.1 Probabilities that a switch be 'antineoclassical'

$\pi(r, R_{sup})$ (*My method based on w(r) curves*)									
$R_{sup} \rightarrow$ / r	*0.5*	*1*	*2*	*5*	*10*	*100*	$\rightarrow\infty$	*D'Ippolito original*	*D'Ippolito corrected*	*D'Ippolito reinterpreted*
0.01	.0692	.0573	.0494	.0441	.0421	.0401	.0399	.0048	.3414	.0653
0.02	.0909	.0778	.0688	.0623	.0597	.0573	.0571	.0092	.3490	.0680
0.03	.1040	.0906	.0812	.0742	.0715	.0689	.0686	.0134	.3562	.0706
0.04	.1130	.0997	.0903	.0830	.0802	.0775	.0772	.0174	.3630	.0730
0.05	.1199	.1068	.0973	.0900	.0871	.0843	.0840	.0212	.3694	.0753
0.06	.1254	.1124	.1030	.0956	.0928	.0900	.0896	.0248	.3755	.0774
0.08	.1339	.1211	.1118	.1045	.1016	.0988	.0985	.0317	.3866	.0814
0.10	.1403	.1276	.1184	.1112	.1084	.1056	.1052	.0349	.3965	.0850
0.12	.1455	.1327	.1237	.1165	.1137	.1109	.1106	.0439	.4054	.0883
0.14	.1501	.1370	.1280	.1209	.1181	.1153	.1150	.0494	.4134	.0912
0.16	.1540	.1406	.1316	.1246	.1219	.1191	.1188	.0546	.4206	.0939
0.18	.1577	.1439	.1348	.1278	.1250	.1223	.1220	.0595	.4271	.0963
0.20	.1612	.1467	.1376	.1305	.1278	.1251	.1248	.0642	.4330	.0985
0.25	.1695	.1529	.1433	.1363	.1336	.1309	.1306	.0747	.4454	.1032

Continued

Table 25.1 Continued

$\pi(r, R_{sup})$ (*My method based on* $w(r)$ *curves*)										
$R_{sup} \rightarrow$ / r	*0.5*	*1*	*2*	*5*	*10*	*100*	$\rightarrow\infty$	*D'Ippolito original*	*D'Ippolito corrected*	*D'Ippolito reinterpreted*
0.30	.1777	.1580	.1479	.1408	.1380	.1353	.1350	.0809	.4551	.1069
0.40	.1976	.1671	.1553	.1476	.1448	.1421	.1418	.0998	.4690	.1123
0.50		.1753	.1612	.1529	.1499	.1470	.1467	.1127	.4780	.1159
0.60		.1837	.1663	.1571	.1539	.1510	.1506	.1235	.4841	.1183
0.80		.2031	.1753	.1639	.1603	.1570	.1566	.1405	.4912	.1212
1.00			.1837	.1695	.1653	.1616	.1612	.1534	.4948	.1227
1.20			.1919	.1742	.1695	.1653	.1649	.1636	.4968	.1236
1.60			.2105	.1824	.1763	.1713	.1708	.1786	.4986	.1244
2.00				.1895	.1819	.1759	.1753	.1891	.4993	.1247
3.00				.2053	.1928	.1844	.1836	.2054	.4998	.1249
4.00				.2216	.2015	.1905	.1895	.2149	.4999	.1249
5.00					.2090	.1952	.1940	.2210	.4999	.125
10						.2093	.2072	.2345	.5	.125
20						.2223	.2189	.2420	.5	.125
30						.2294	.2248	.2446	.5	.125

Part I. D'Ippolito's logical slip

In the model studied by Samuelson (1962), Hicks (1965), Garegnani (1970) and D'Ippolito (1987), to be called in what follows the Samuelson-Garegnani model, a single consumption good can be produced via different techniques, each one requiring labour and a *different* circulating capital good, with the capital good in turn produced by itself and labour, both processes requiring one period for their completion. (Thus the transition from one technique to the other is not studied; the exercises consist of comparisons of long-period positions in which the transitional technologies do not appear.) The production processes last one period. There are constant returns to scale; α_i, β_i are the technical coefficients, respectively, of capital good of type i and of direct labour in the production of the consumption good according to technique i; a_i, b_i are the technical coefficients respectively of the capital good and of direct labour in the production of capital good of type i. For simplicity each capital good will be measured here in such units that its production needs one unit of direct labour, so $b_i = 1$, for all i. The consumption good is the numéraire. Wages are paid at the end of the production period. The price-of-production equations for any technique i are, with w the rate of wages in terms of the consumption good and p_i the price of the capital good:

$$1 = \alpha_i p_i(1 + r) + \beta_i w \tag{25.1}$$

$$p_i = a_i p_i(1 + r) + w. \tag{25.2}$$

In what follows the subscript i that specifies the technique will be omitted when unnecessary.

The technical coefficients are non-negative. These equations establish a monotonic negative functional dependence of w on r:

$$w = [1 - (1 + r)a]/[\beta + (1 + r)(\alpha - a\beta)] \tag{25.3}$$

such that, as long as $a > 0$ and that direct or indirect labour is necessary to produce one unit of net product (consisting of the consumption good), this function crosses the non-negative orthant with negative slope and positive intercepts on both axes, determined by:

$$r(0) = R = (1 - a)/a \tag{25.4}$$

$$w(0) = W = (1 - a)/[\beta + (\alpha - a\beta)], \tag{25.5}$$

I shall call $w(r)$ *curve* this nonnegative portion of the function $w(r)$ defined by (25.3). In the figures the rate of profit is measured on the abscissa.

It is known that in this model two $w(r)$ curves can cross each other in the non-negative orthant at most twice. If they do cross twice, the switch point at the higher level of r gives rise to reverse capital deepening.

Given a $w(r)$ curve, it is known that, having selected a point (r, w) on this curve, as long as the economy is stationary (which will be assumed here) the value of capital per unit of labour is given by the absolute value of the slope of the straight line connecting this point with the point $(0, W)$. This is because in a stationary economy the net product per unit of labour consists solely of the consumption good and equals W; at long-period prices it must be true that the net product distributes itself between wages and profits (or interest) i.e., with k the value of capital per unit of labour, $W = w + rk$, which can be re-written as

$$k = (W - w)/r. \tag{25.6}$$

Therefore if at r^* there is a switch between two techniques, and if we call technique 1 the one dominant for r slightly less than r^* and technique 2 the one dominant for r slightly greater than r^*, then the switch gives rise to reverse capital deepening, i.e. $k_2(r^*) > k_1(r^*)$, if and only if $W_2 > W_1$, cf. Figure 25.1.

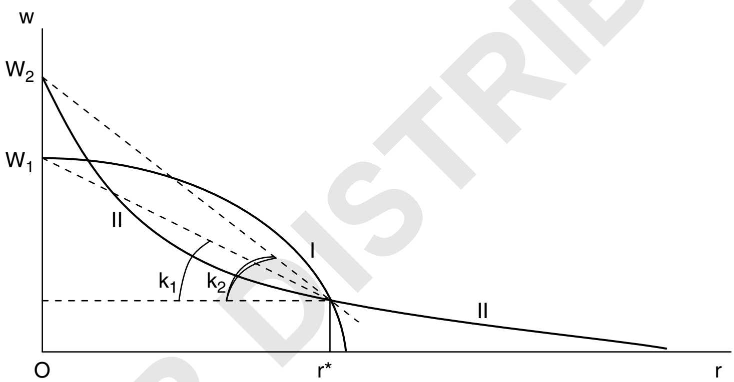

Figure 25.1 Derivation of the value of capital per unit of labour from the $w(r)$ curves

For this model, D'Ippolito tries to determine the 'a priori' probability $P_{me}(r)$ that, *if two techniques have a switchpoint at a rate of profit equal to r, this switchpoint be associated with reverse capital deepening* or, as he unscientifically puts it, be 'perverse' (I will call it an 'antineoclassical' switchpoint). He too calls technique 2 the one which becomes dominant to the *right* of the switchpoint, i.e. by assumption 25.

$$-dw_2(r)/dr \equiv -w_2'(r) < -dw_1(r)/dr \equiv -w_1'(r)$$

where r is the rate of profits at the switchpoint. To determine $P_{me}(r)$ he proceeds as follows. Because $w_1(r) = w_2(r)$ the previous inequality can be re-written

$$-w_2'(r)/w_2(r) < -w_1'(r)/w_1(r). \tag{25.7}$$

Since

$$w'(r) \equiv dw(r)/dr \tag{25.8}$$
$$= \{-a[\beta + (1+r)(\alpha - a\beta)] - [1-(1+r)a](\alpha - a\beta)\}/[\beta + (1+r)(\alpha - a\beta)]^2,$$

and

$$p = 1/[\beta + (1+r)(\alpha - a\beta), \tag{25.9}$$

equation (25.7) simplifies to

$$\alpha_2 p_2/(1-(1+r)a_2) \le \alpha_1 p_1/(1-(1+r)a_1). \tag{25.10}$$

In order for the switch to be an ‘antineoclassical’ one, the (value of) capital per unit of labour at the switchpoint must be greater for technique 2 than for technique 1. D’Ippolito notices that, as techniques switch, capital per unit of labour k, and capital per unit of net output K, vary in the same direction,[6] so this condition can be written $K_2(r) > K_1(r)$, or, since $K = \alpha p/(1-a)$ (because $\alpha/(1-a)$ units of the capital good are employed in a stationary economy producing 1 unit of the consumption good as net product):

$$\alpha_2 p_2/(1-a_2) \le \alpha_1 p_1/(1-a_1). \tag{25.11}$$

D’Ippolito puts

$$v \equiv \alpha_1 p_1/(\alpha_2 p_2),\ \rho \equiv 1+r \tag{25.12}$$

and re-writes inequalities (25.10), (25.11) as:

$$a_2 - 1/\rho \le (1/v)(a_1 - 1/\rho). \tag{25.13}$$

$$a_2 - 1 > (1/v)(a_1 - 1). \tag{25.14}$$

Let us then represent the points (a_1, a_2) on the non-negative orthant of a plane (Figure 25.2). Assume a given r and a given v. Equation (25.13) implies $a_2 \le a_1/v - 1/(v\rho) + 1/\rho$, i.e. a_2 must be below the straight line ‘S’ with slope $1/v$ and passing through the point $(1/\rho, 1/\rho)$. Equation (25.14) implies $a_2 > a_1/v - 1/v + 1$, i.e. a_2 must be above the straight line ‘E’ with slope $1/v$ and passing through the point $(1, 1)$. On the other hand neither a_1 nor a_2 can be greater than $1/\rho$ if w is to be non-negative, because the maximum rate of profits is $R = 1/a - 1$. Hence the set of couples (a_1, a_2) which satisfy both (25.13) and (25.14) is the set of the points, internal to the square OCBQ (of side length equal to $1/\rho$), which are both to the right of the line ‘S’ (i.e. AB) and to the left of the line ‘E’ (i.e. A’E) in Figure 25.2. This set is not empty only if $v < 1$, because if $v > 1$ line ‘S’ is to the right of line ‘E’.[7] It is the shaded area F in Figure 25.2a.

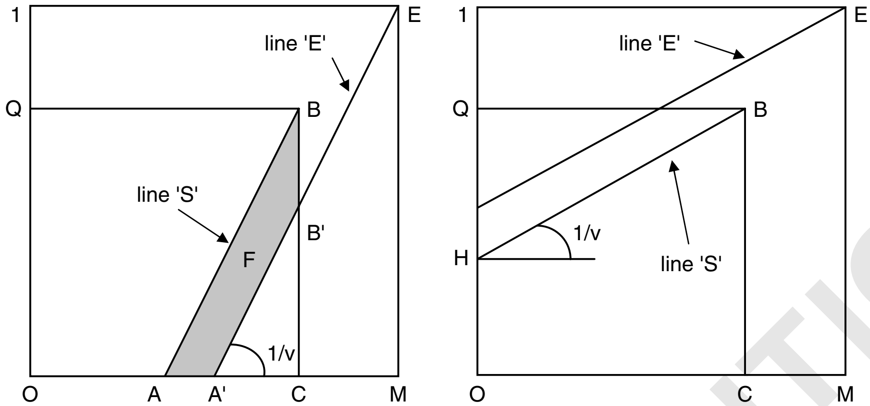

Figure 25.2a&b

Having reached this pleasant graphical result, D'Ippolito proceeds in a way which appears marred by a logical slip.

He argues (D'Ippolito, 1987, p. 17) that, for given r and v, the probability that the switch be 'antineoclassical' is given by the ratio between the surface of the area F of points satisfying both constraints, and the surface $1/\rho 2$ of the whole square OCBQ, i.e. is given by $\rho 2$ times $F(r, v)$ (if with $F(r, v)$ one indicates the surface of F).

This is difficult to accept. Let us concede to D'Ippolito for the sake of argument the right to limit the inquiry to the coefficients a_1, a_2, and to assume, as he clearly does, that all points (a_1, a_2) compatible with the given r might occur with equal probability.[8] But D'Ippolito forgets that he has *assumed* that the two techniques have a switchpoint at the given rate of profits r, and that, since he has *decided* to call technique 2 the one which becomes dominant to the right of r, then (a_1, a_2) satisfies (25.13) by assumption; the points above line 'S' are therefore out of the question. So for given r and v, by assumption (a_1, a_2) is in the triangle ABC of Figure 25.2a if $v < 1$, or in the trapeze OCBH of Figure 25.2b if $v > 1$. Then the correct ratio – I will call it $Z(r, v)$ – between area of 'antineoclassical' cases and area of possible cases, for a given r and a given $v < 1$, is the ratio between $F(r, v)$, and the surface – I shall call it $D(r, v)$ – of the triangle ABC of Figure 25.2a, while it is zero if $v \geq 1$.

Under the assumption, which D'Ippolito explicitly makes (ibid., p. 18), that all values of v are equiprobable,[9] one may therefore proceed to calculate the average probability $Z^*(r)$ that at a given r a switch be 'antineoclassical' under the assumption that v is random but < 1, by integrating $Z(r, v)$ with respect to v from 0 to 1. $Z^*(r)$ is not the average probability $P_{me}(r)$ that at a given r a switch be 'antineoclassical', because it is determined under the assumption that $v < 1$ so it leaves out the possibility that $v > 1$. But D'Ippolito says that the case $v > 1$ 'would cover the remaining 50% of cases' (ibid., p. 16), so he appears to authorize us to assume that the probability that $v < 1$ is 50%. Then $P_{me}(r)$ is simply one half of $Z^*(r)$.

D'Ippolito, on the contrary, having said that the probability of an 'antineoclassical' switch, for a given r and a given $v < 1$ is given by $\rho^2 F(r, v)$, simply goes on to integrate this probability over v from 0 to 1 in order to obtain his $P_{me}(r)$, without mentioning any more the fact that there is also the case $v > 1$. I have been unable to find a way to make his several statements consistent.[10] His probabilities are reported in column 9 of Table 25.1, under the heading 'D'Ippolito original'. They converge to 25% as r tends to $+\infty$.

The calculation of the probability that a switch be 'antineoclassical' with the correction I find necessary, i.e. $P_{me}(r) = Z^*(r)/2$, yields much higher values than the ones calculated by D'Ippolito; they are listed in column 10 of Table 25.1 under the heading 'D'Ippolito corrected'. They converge to 50% as r tends to $+\infty$.

Appendix A shows how to determine the surfaces, whose ratios determine the probability of an antineoclassical switch for given (r, v) according to D'Ippolito, and according to my correction.

Perhaps D'Ippolito thought that he had the right to neglect to consider the cases $v > 1$ because in some way he was already taking the existence of those cases into account, by dividing $F(r, v)$ by the whole surface of the square OBCQ instead of by the sole surface $D(r, v)$.[11] But then a more consistent estimation procedure would appear to be the following one.

Let us drop the assumption that all values of $v < 1$ are equiprobable, by noticing that $v < 1$ and $v > 1$ are symmetrical and hence equiprobable only if no constraint on the coefficients is added so that all points in OBCQ might be picked by the random technique selection process; while here there *is* a constraint, and this is that at the switchpoint it is technique 2 which becomes dominant to the right of r. This constraint makes only the points in ABC eligible, i.e. a fraction of OCBQ which is the smaller, the smaller is v. If one then considers all values of a_1, a_2 as equally probable,[12] one may conclude that the values of v are not all equiprobable, but are the more probable, the greater the portion of OCBQ that makes the switch possible. The natural assumption then is to assume that the probability of each value of v, or of each value of $1/v$ if $v > 1$,[13] is proportional to the ratio between $D(r, v)$ and the area of OCBQ.

In other words, let us replace v in $Z(r, v)$ with a variable y, $0 \leq y \leq 2$, defined as $y = v$ if $v \leq 1$ and $y = 2 - (1/v)$ if $v > 1$. The density function of y, $p(y)$, is assumed linear, going from 0 to 1 as y goes from 0 to 2, corresponding to the ratio $D(r, y)/(1/\rho^2)$ between the surface of ABC or of OCBH, and the surface of OCBQ. Then the average probability $P_{me}(r)$ that a switch be antineoclassical is the definite integral of $Z(r, y)p(y)$ over y from 0 to 2, divided by 2; but since $Z(r, y) = 0$ for $y > 1$, it suffices to calculate the definite integral of $Z(r, v)p(v)$ over v from 0 to 1, and then divide by 2. Since for $v < 1$ it is $D(r, v) = v/(2\rho^2)$ (cf. Appendix A) then $p(v) = v/2$, so $P_{me}(r)$ is one half of the definite integral of $vZ(r, v)$ over v from 0 to 1.

The resulting probabilities are listed in the last column of Table 25.1, under the heading 'D'Ippolito reinterpreted'. I do not claim that this 'reinterpretation' has strong textual support.

Part II. A different approach

Now I explore a different method of estimating $P_{me}(r)$. This method starts by assuming an initially given switchpoint $C = (r^*, w^*)$ in the (r, w) plane in which one draws the $w(r)$ curves of the different techniques.

The form of a $w(r)$ curve in this model depends on α, β, a. There are therefore three degrees of freedom. If we establish that the curve must pass through C and must have a given $R \geq r^*$ (and $> r^*$ if $w^* > 0$) and a given $W \geq w^*$ (and $> w^*$ if $r^* > 0$), the three degrees of freedom are eliminated and the $w(r)$ curve, i.e. the technique, is completely determined. The intuition behind what follows is that, by assuming an equal probability of all values of r between r^* and R, and of w between w^* and W, one must be able to determine the probability that two curves that switch at C have a second switch for a lower r so that the switch at C is antineoclassical. For brevity below I drop the asterisks.

Not all quadruplets (w, r, R, W) correspond to an economically acceptable technique (D'Ippolito 1987, p. 34). For a given point C and a given a, the values of α and β must satisfy equations (25.3) and (25.5) where w, r and a are given; this can be re-written as

$$w(1 + r)\alpha + w[1 - (1 + r)a]\beta = 1 - (1 + r)a \tag{25.15}$$

$$W\alpha + W(1 - a)\beta = 1 - a. \tag{25.16}$$

For a given W, this is a system of two linear equations in α, β. The solutions are

$$\beta = \frac{w(1+r)(1-a) - W\left[1-(1+r)a\right]}{wWr} \tag{25.17}$$

$$\alpha = \frac{(W - w)\left[1-(1+r)a\right](1-a)}{wWr} \tag{25.18}$$

As long as $wWr > 0$, for β to be non-negative the numerator on the right-hand side of (25.9) must be non-negative i.e.

$$w(1 + r)(1 - a) \geq W[1 - (1 + r)a] \tag{25.19}$$

which, since $(1 - a)/a = R$ implies $[1 - (1 + r)a]/(1 - a) = (R - r)/R$, can be re-written as

$$W \leq w(1 + r)\frac{R}{R - r} \equiv W_{max} \tag{25.20}$$

The right-hand side of this inequality defines a new variable W_{max} whose meaning will be now clarified. Expression $w/(R - r)$ is the absolute slope of the straight line connecting C with R (the point where the $w(r)$ curve touches the horizontal axis),

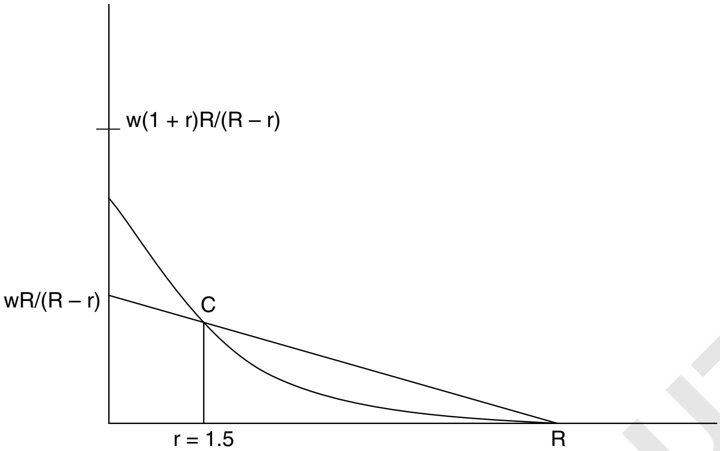

Figure 25.3

so $wR/(R - r)$ is the value of w where this straight line crosses the vertical axis; see Figure 25.3.

Therefore W cannot exceed a value $W_{max}(r; w, R)$ determined by the value of this point multiplied by $(1 + r)$.

This constraint $W \leq W_{max}$ is the sole constraint besides $W > w$ (assuming $r > 0$) and $R > r$: $\alpha \geq 0$ does not pose a constraint because $W - w \geq 0$ and $[1 - (1 + r)a](1 - a) = (R - r)/R \geq 0$; as to R, it can be chosen arbitrarily close to r by increasing a; and W can be chosen arbitrarily close to w through the sufficiently high values of α and β determined by equations (25.17) and (25.18).

The $w(r)$ curve will be concave (downwards) if W is below the point $W_{max}/(1 + r) = wR/(R - r)$ where the straight line through R and C crosses the vertical axis; it will be convex if W is in between this point and the point of ordinate W_{max}.

For given points C (with $r > 0$) and R, therefore, a given W satisfying the constraint $w < W \leq W_{max}$ uniquely determines the $w(r)$ curve. This means that all the possible $w(r)$ curves passing through given points $C = (r, w)$ and R can be generated by letting W vary in the interval (w, W_{max}).

It will be assumed that each value of W in this interval has the same probability. This does not appear to be a more arbitrary assumption than the analogous ones in D'Ippolito's analysis.

With C still fixed let us now suppose that no available technique, of those whose curve passes through C, has an associated R greater than a certain finite value R_{sup}.

I find such an assumption (which does not prevent one from fixing a very high R_{sup}, nor from admitting that technical progress increases R_{sup}) more reasonable than the assumption (implicit in D'Ippolito) that R can take any value, however great: the latter assumption would imply that one can get as near as one likes to producing with unassisted labour. Anyway my assumption is not necessary to the method proposed here; it can be seen as only an intermediate step to assuming $R_{sup} = +\infty$.

Let us suppose that two (admissible) couplets (R, W) are repeatedly randomly selected, thus randomly selecting two $w(r)$ curves through C and hence two techniques. The probability that the two values of R coincide is zero, so let us assume that they differ, and let us now call technique 1 the one associated with the lower R, and technique 2 the other one. Hence $R_1 < R_2$ by assumption. Note that technique 2 is no longer defined as the one which is dominant to the right of the switchpoint, but as the one with the higher R.

The probability that $W_1 = W_2$ is analogously zero. A necessary condition for there to be a second intersection of the two $w(r)$ curves either to the left or to the right of C, is that $W_1 < W_2$; see Figures 25.4a and 25.4b.

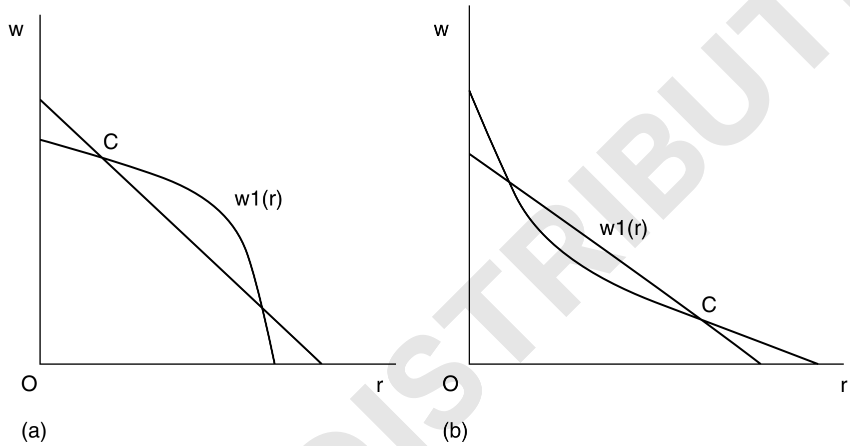

Figure 25.4

This condition is also not quite sufficient, because it is possible that the two $w(r)$ curves be tangent in C (see Figure 25.5c); but if W_1 and W_2 are, as we are assuming, continuous variables, the probability of this case is zero: once r, w, R_2, W_2 and $R_1 < R_2$ are assigned, the condition that the two $w(r)$ curves be tangent in C uniquely determines W_1, as will be shown later, and the probability of such a precise value of W_1 is zero. Thus this case can be neglected. Therefore except for this negligible fluke, $W_1 < W_2$ is a necessary and sufficient condition for the two $w(r)$ curves to reswitch.

The probability that $W_1 < W_2$, to be indicated with the symbol μ^*, is the probability that there be a second switchpoint, conditional on the curves passing through the assigned point C. Since this second switchpoint might be to the right of C, in which case the switch in C does not give rise to reverse capital deepening (i.e. is not 'antineoclassical'), we must also determine the probability – to be indicated with the symbol P – that, in case the two curves reswitch, the second switch be to the left of C; then the probability that the switch in C be 'antineoclassical' will be given by the product μ^*P and will be indicated with the symbol $\pi(r, R_{sup})$ because it will be shown that it depends only on the values of r and R_{sup}.

Let us now determine the probability μ^* that, having assigned the switch-point $C = (r; w)$ with $r, w > 0$, having randomly selected two $w(r)$ curves passing through C, and having called 'technique 2' the one with the greater R, it is $W_1 < W_2$.

Let R_1 and R_2 be initially given, and let us consider the admissible intervals for W_1 and W_2:

$$w < W_1 \leq w(1+r)\,\frac{R_1}{R_1 - r} \equiv W_{1max} \tag{25.20}$$

$$w < W_2 \leq w(1+r)\,\frac{R_2}{R_2 - r} \equiv W_{2max} \tag{25.21}$$

Obviously $(W_{2max} - r) < (W_{1max} - r)$ because W_{max} decreases as R increases with C fixed. So it is possible that $W_1 > W_{2max}$ but it is excluded that $W_2 > W_{1max}$.

If we consider all values of W_i as equiprobable within its admissible interval, then the probability, that $W_1 < W_2$ conditional on $W_1 \leq W_{2max}$, is 1/2; while the probability that $W_1 < W_2$ conditional on $W_1 > W_{2max}$ is zero; so the unconditional probability that $W_1 < W_2$ must be 1/2 times the ratio, to be indicated as Q, between $(W_{2max} - w)$ and $(W_{1max} - w)$. This ratio is given by:

$$\begin{aligned} Q &\equiv (W_{2max} - w)/(W_{1max} - w) \\ &= \{[(1+r)wR_2/(R_2 - r)] - w\}/\{[(1+r)wR_1/(R_1 - r)] - w\} \\ &= [(R_1 - r)/(R_2 - r)]/[(R_1 + 1)/(R_2 + 1)]. \end{aligned} \tag{25.22}$$

Q comes out not to depend on w; the probability, that $W_1 < W_2$ for an assigned quadruple (w, r, R_1, R_2), remains the same if only w is varied (i.e. if the point C is moved vertically). So from now on we forget about w.

Let us now suppose that only r and R_2 are assigned, and let us determine the average probability $\mu = \mu(r, R_2)$ (which is not yet μ^*!) that $W_1 < W_2$ as R_1 is made to vary from r to R_2.

In order to reach an intuitive grasp, let us start by noticing that Q tends to zero as R_1 tends to r, and tends to 1 as R_1 tends to R_2. For each assigned R_1 the probability that $W_1 < W_2$ is $Q/2$, so it varies from 0 to 1/2 as R_1 is made to vary from r to R_2. If at the denominator of Q, instead of $(R_1 + 1)/(R_2 + 1)$, there were 1, then Q would indicate the proportion of the distance between r and R_2 travelled by R_1, so it would increase linearly from 0 to 1, its mean would be 1/2, and so the average probability μ would be 1/4 if one considers all values of R_1 between r and R_2 as equiprobable. That the denominator of Q is on the contrary always less than 1 except when $R_1 = R_2$ means that μ must be greater than 1/4; furthermore since $Q/2$ cannot be greater than 1/2 (because for given r and R_2, Q is a strictly increasing function of R_1 as shown by its derivative, and it tends to 1 as R_1 tends to R_2), μ cannot be greater than 1/2.

Formally,

$$\mu = \frac{1}{2(R_2 - r)}\int_r^{R_2} \frac{(R_1 - r)(R_2 + 1)}{(R_1 + 1)(R_2 - r)} dR_1 \tag{25.23}$$

$$= 1/2 \cdot (R_2 + 1)[R_2 - r \cdot ln(R_2 + 1) + r \cdot ln(1 + r) + ln(1 + r) - r]/(r - R_2)^2.$$

It will be useful to notice that this is an increasing function of R_2, because its derivative is

$$\partial\mu/\partial R_2 = -1/2 \cdot (1 + r)\{(r + R_2)[ln(R_2 + 1) - ln(r + 1)] + 2[r - ln(1 + r)] - 2[R_2 - ln(1 + R_2)]\}/(-R_2 + r)^3 \tag{25.24}$$

always positive for $0 < r < R_2$, because, since the denominator is negative, the sign of $\partial\mu/\partial R_2$ depends on the sign of

$$(r + R_2)[ln(R_2 + 1) - ln(r + 1)] + 2[r - ln(1 + r)] - 2[R_2 - ln(1 + R_2)]; \tag{25.25}$$

this expression reduces to zero if $R_2 = r$, where its first and second derivatives with respect to R_2 are zero, while the third derivative, which is $(2r - R_2 + 1)/(R_2 + 1)^3$, reduces to $1/(1 + r)^2 > 0$ if $R_2 = r$. Therefore when $R_2 = r$, expression (25.25) is zero but is an increasing function of R_2, so to the right of $R_2 = r$ expression (25.24) is positive. And since the second derivative of expression (25.25) with respect to R_2 is $(R_2 - r)/(R_2 + 1)^2$ which is always positive for $R_2 > r$, the first derivative is always an increasing function of R_2 and so it too is positive to the right of $R_2 = r$, and therefore expression (25.25) too is increasing and therefore positive to the right of $R_2 = r$, which completes the proof.

The limit of μ for R_2 tending to r from the right is 1/4, for R_2 tending to $+\infty$ is 1/2, for r tending to zero is

$$\lim_{r \to 0_+} \mu = \frac{R_2 + 1}{2R_2^2} \cdot (R_2 - ln(R_2 + 1)).$$

Having determined $\mu(r, R_2)$ and having found how it varies with R_2, the average probability $\mu^*(r, R_{sup})$ that $W_1 < W_2$ when R_2 is also free to vary on an interval (r, R_{sup}) is simply the definite integral of $\mu(r, R_2)$ over R_2 from r to R_{sup}.

Through e.g. Maple one can obtain the exact analytical expression for $\mu^*(r, R_{sup})$ as just defined (it is very long and for this reason omitted here). But for the purpose of arriving at numerical estimates of μ^* also for very high values of R_{sup}, and in the limit for R_{sup} tending to $+\infty$, it seems preferable to replace the assumption, of a uniformly distributed probability of all values of R between r and R_{sup}, with the assumption of a uniform probability distribution of all values of the coefficient $a = 1/(1 + R)$ in the corresponding interval, because, as we let R_{sup} tend to $+\infty$, the first assumption would result in a probability tending to zero of all values of R in any finite interval, i.e. in terms of the coefficient a we would be assigning, in the limit, probability zero to all nonzero values of this coefficient, which is absurd. There is of course some arbitrariness in assuming a uniform probability distribution of all admissible values of a, but some such arbitrariness appears inevitable in this kind of exercises.

Replacing then R_1 and R_2 with $(1 - a_1)/a_1$ and $(1 - a_2)/a_2$ in the equations from (25.20) to (25.23) we have:

$$W_{1max} = w(1 + r)(1 - a_1)/(1 - a_1 - ra_1) \tag{25.26}$$

$$W_{2max} = w(1 + r)(1 - a_2)/(1 - a_2 - ra_2) \tag{25.27}$$

$$\begin{aligned} Q/2 &\equiv (W_{2max} - w)/[2(W_{1max} - w)] \qquad (25.28)\\ &= \{w(1 + r)(1 - a_2)/(1 - a_2 - ra_2)\}/2\{w(1 + r)(1 - a_1)/(1 - a_1 - ra_1)\}\\ &= ½ \cdot[(1 - a_1 - ra_1)(1 - a_2)] / [(1 - a_2 - ra_2)(1 - a_1)]. \end{aligned}$$

$Q/2$ as here defined is the probability that $W_1 < W_2$, i.e. that there is a second switchpoint, once r, a_1 and a_2 are assigned.

Now μ is re-defined as the average probability that $W_1 < W_2$ as a_1 varies from a_2 to $1/(1 + r)$, equal to the definite integration of $Q/2$ over a_1 from a_2 to $1/(1 + r)$:

$$\mu = \frac{1/2}{\left[\frac{1}{1+r} - a_2\right]} \int_{a_2}^{1/(1+r)} \frac{(1 - a_1 - ra_1)(1 - a_2)}{(1 - a_2 - ra_2)(1 - a_1)} da_1 \,. \tag{25.29}$$

And $\mu^*(r, R_{sup})$ is given by the following definite integral:

$$\mu^* = \frac{1}{\frac{1}{1+r} - \frac{1}{1+R_{\text{sup}}}} \int_{1/(1+R_{\text{sup}})}^{1/(1+r)} \mu da_2 \,. \tag{25.30}$$

The analytical solution of these integrals is very complex and in the end unnecessary. The function at the bottom of these integrations is $Q/2$ as defined by equation (25.28), which is a smooth, well-behaved function, so one can legitimately approximate the calculation of these integrals with the method of rectangles. I have used Maple (more precisely, an old MapleV Release3 Student Edition version) to approximate μ and μ^* through the area of 40 rectangles of equal basis and of height equal to the value of the function in their middle point. Maple determines the analytical expressions for this approximation, and substituting into it the assigned values of r and of R_{sup} one obtains the probabilities that there be a second switchpoint. The numerical values of these probabilities are listed in Table 25.2.

Table 25.2 μ^*, i.e. probability, according to my method based on the $w(r)$ curves, that two w(r) curves which intersect have a second intersection

r	R_{sup}						
	0.5	*1*	*2*	*5*	*10*	*100*	$\rightarrow \infty$
0.01	.4227	.4374	.4466	.4530	.4553	.4575	.4578
0.02	.3934	.4114	.4231	.4314	.4345	.4374	.4377
0.03	.3741	.3938	.4068	.4162	.4197	.4231	.4235
0.04	.3597	.3804	.3943	.4044	.4082	.4119	.4123
0.05	.3484	.3697	.3841	.3947	.3987	.4026	.4030
0.06	.3391	.3607	.3755	.3865	.3907	.3947	.3952
0.08	.3246	.3465	.3617	.3732	.3776	.3818	.3823
0.10	.3135	.3354	.3509	.3627	.3672	.3716	.3721
0.12	.3047	.3265	.3421	.3540	.3586	.3631	.3636
0.14	.2976	.3192	.3347	.3468	.3514	.3559	.3565
0.16	.2915	.3129	.3284	.3405	.3452	.3498	.3503
0.18	.2864	.3076	.3230	.3351	.3398	.3444	.3449
0.20	.2819	.3028	.3182	.3303	.3350	.3396	.3401
0.25	.2730	.2933	.3084	.3204	.3251	.3297	.3303
0.30	.2662	.2859	.3008	.3127	.3174	.3219	.3225
0.40	.2566	.2753	.2897	.3013	.3059	.3105	.3110
0.50		.2680	.2819	.2933	.2978	.3023	.3028
0.60		.2626	.2761	.2872	.2917	.2961	.2966
0.80		.2550	.2680	.2787	.2830	.2874	.2879
1.00			.2626	.2730	.2772	.2814	.2819
1.20			.2586	.2688	.2730	.2771	.2776
1.60			.2534	.2633	.2673	.2713	.2718
2.00				.2596	.2636	.2675	.2680
3.00				.2545	.2583	.2621	.2625
4.00				.2517	.2554	.2592	.2596
5.00					.2537	.2574	.2578
10						.2536	.2540
20						.2516	.2521
30						.2510	.2514

We must now determine P, the probability that, assuming there is another switchpoint between the two randomly selected $w(r)$ curves passing through a given point C, this second switchpoint is to the left of C. (P too will come out to be independent of w.)

Let us initially take as given not only the point C but also the $w_2(r)$ curve i.e. R_2 and W_2. For each R_1 such that $r < R_1 < R_2$, there is a unique value of W_1 such that the two $w(r)$ curves are tangent in C. Let W_1^(R_1) indicate this value of W_1. W_1^ is determined by considering W_1 as a variable in equation (25.5) in the system of equations (25.3) – (25.4) – (25.5) applied to technique 1, and adding, first, equations (25.3), (25.4), (25.5) applied to technique 2 with W_2 given, and second, the condition that in C the slopes of the two $w(r)$ curves must be the same i.e.

$$-\alpha_1/(-\alpha_1 - \alpha_1 \cdot r - \beta_1 + a_1 \cdot \beta_1 + a_1 \cdot \beta_1 \cdot r)^2$$
$$= -\alpha_2/(-\alpha_2 - \alpha_2 \cdot r - \beta_2 + a_2 \cdot \beta_2 + a_2 \cdot \beta_2 \cdot r)^2 \,. \tag{25.31}$$

The uniqueness of W_1^(R_1) derives from the fact that to each triplet of points (C, R, W) there corresponds a unique $w(r)$ curve and that, for given C and R, the convexity of the $w(r)$ curve monotonically increases with W, passing from initially concave to straight to convex: so also the absolute value of the slope in C of the $w(r)$ curve monotonically increases with W; therefore there will be a unique value of W_1 making the slope of $w_1(r)$ in C equal to the assigned slope of the $w_2(r)$ curve. Indeed let us prove that for given r and a the absolute value of the slope of a $w(r)$ curve is an increasing function of W. In:

$$dw/dr = -\alpha/[\beta + (1 + r)(\alpha - a\beta)] \tag{25.32}$$

let us replace α and β with the expressions determining them in equations (25.17) and (25.18); simplifying we obtain:

$$-\, dw/dr = \{w(Wa - wa - W + w)\}/\{Wr(a - 1 + ar)\} \tag{25.33}$$

whose derivative is:

$$d^2r/dr^2 = w^2(1 - a) \,/\, W^2r[1 - a(1 + r)] > 0 \tag{25.34}$$

because the numerator is positive ($0 < a < 1$ if $R > 0$), and the denominator is positive because $0 < a(1 + r) < 1$, owing to $a = 1/(1 + R)$ and therefore $a(1 + r) = (1 + r)/(1 + R) < 1$.

(Using a here instead of R has the same motivation as in equations (25.26) and ff.) W_1^ is a function of $a1$, $a2$, r, w, W_2:

$$W_1\text{^} = W_2 \cdot w(-a_1 + a_1 \cdot a_2 + ra_1a_2 + 1 - a_2 - ra_2)/$$
$$(-rW_2a_2 + wa_1a_2 - wa_2 + wra_1a_2 + a_1rW_2 - wa_1 + w - wra_1) \tag{25.35}$$

Since $w(r)$ is a hyperbola, as R_1 increases continuously in the interval (r, R_2) also W_1^ increases continuously and goes through all values in the interval (w, W_2). Thus, for a given R_1, if $W_1 < W_1$^ then the slope in C of $w_1(r)$ is less (in absolute value) than the slope of $w_2(r)$, so the second switchpoint is to the right of C; if $W_1 > W_1$^, the second switchpoint is to the left of C. The probability that the switch in C be 'antineoclassical' depends therefore on the probability that $W_1 > W_1$^. We may assume that this probability is to 1 like the length of the interval (W_1^, W_2) is to the length of the interval (w, W_2), and therefore that it is equal to ($W_2 - W_1$^)/($W_2 - w$).

(For W_2 and/or w tending to $+\infty$ it might seem that the same problem arises, which earlier induced me to replace R with a; but this problem will not arise because W_2 and w will disappear from the formulas through simplification.)

For R_1 tending to R_2 this ratio tends to zero, and it tends to 1 for R_1 tending to r. But how it varies within the interval (r, R_2) is a complicated thing and for the calculation of the average value of ($W_2 - W_1$^)/($W_2 - w$), when both W_2, R_1 (or rather a_1), and R_2 (or rather a_2) are considered random variables with a uniform probability distribution within the respective admissible intervals, Maple has been again indispensable.

For given values of r, w, R_2 (or rather a_2), the values of W_2 can vary in the interval (w, W_{max}). It will be assumed that all values of W_2 in this interval are equiprobable. The probability, that for given r, w, R_2 and R_1 one finds that $W_1 >$ W_1^, is given by the definite integral:

$$\frac{1}{W_{2\max} - w} \int_{w}^{W_{2\max}} \frac{W_2 - W_1\hat{}}{W_2 - w} dW_2 \qquad (25.36)$$

where w is given, $W_{2max} = w(1 + r)R_2/(R_2 - r) = w(1 + r)(1 - a_2)/(1 - a_2 - ra_2)$, and W_1^ is given by equation (25.35) and is therefore a function of r, w, a_1, a_2 and W_2. This probability is a function of r, w, a_1 and a_2.

It is useful to reach this same probability in another way. Let x be a scalar, variable between 0 and 1, and, in the expression ($W_2 - W_1$^)/($W_2 - w$) let us replace W_1^ with its value given by equation (25.27), and let us replace W_2 with its expression in terms of w, W_{max} and x, i.e. with:

$$W_2 = w + x[w(1 + r)(1 - a_2)/(1 - a_2 - ra_2) - w]. \qquad (25.37)$$

(The expression inside the square brackets on the right-hand side of (25.37) is $W_{max} - w$; so as x varies from 0 to 1, W_2 varies from w to W_{max}.) If we perform these two substitutions in the expression ($W_2 - W_1$^)/($W_2 - w$), w is eliminated and we obtain an expression for ($W_2 - W_1$^)/($W_2 - w$) which we shall indicate as PA:

$$PA \equiv (W_2 - W_1\hat{})/(W_2 - w) = r(a_2 - a_1 - a_2^2 - ra_2^2 + xra_2 - xra_1 + ra_1 \cdot a_2 + a_1 \cdot a_2)/(2ra_2 - 2a_1a_2 - 2ra_1a_2 - 2ra_2^2 - r^2a_2^2 + a_2^2a_1 + 2ra_2^2a_1 + r^2a_2^2a_1 + xr^2a_2 - xr^2a_1 + a_1 + 2a_2 - 1 - a_2^2). \qquad (25.38)$$

PA indicates the probability that, if two $w(r)$ curves cross in *C*, this switchpoint is 'antineoclassical', when r, a_1, a_2 and W_2 (i.e. x) are assigned. (As announced, W_2 and w have disappeared.) *PA* is the basic function in what follows.

Since x varies between 0 and 1, expression (25.36) becomes:

$$PX \equiv \int_0^1 PA\,dx \tag{25.39}$$

PX is the average probability that the switch in *C* is 'antineoclassical' if there is another switchpoint, when r, a_1 and a_2 are given while W_2 is random between w and W_{2max}. *PX* is independent of w, like *PA*; it is a function of r, a_1 and a_2. Again it can be calculated by approximation.

Now let us consider only r and a_2 as assigned and let us suppose all values of a_1 in the interval $(a_2, 1/(1 + r))$ to be equiprobable; by integrating *PX* with respect to a_1 on the interval $(a_2, 1/(1 + r))$ and dividing by $[1/(1 + r)] - a_2$, we now determine the average probability that the switch in *C* is 'antineoclassical' if there is another switchpoint, when only r and a_2 are assigned. Let this probability be indicated as *PA*1:

$$PA1 \equiv \frac{1}{\frac{1}{1+r} - a_2} \int_{a_2}^{1/(1+r)} PX\,da_1 \tag{25.40}$$

Lastly, let $a_{2inf} \equiv 1/(1 + R_{sup})$ be the minimum possible value of a_2 (and of a_1), i.e. the one corresponding to the assigned R_{sup}. Let us integrate *PA*1 with respect to a_2 on the interval $(a_{2inf}, 1/(1 + r))$ and divide by the length of this interval; in this way we obtain the average probability *P* that the switch in *C* be the 'antineoclassical' one if there is another switchpoint, with only r and R_{sup} given:

$$P(r; R_{sup}) \equiv \frac{1}{\frac{1}{1+r} - a_{2\mathrm{inf}}} \int_{a_{2\mathrm{inf}}}^{1/(1+r)} PA1\,da_2 \quad \text{where } a_{2inf} = (1 + R_{sup})^{-1}. \tag{25.41}$$

MapleV Release 3 Student Version again determines without difficulty the values of these integrals by approximating them with the method of rectangles (again I have used 40 rectangles). The basic function *PA* is very 'regular' so the approximations are certainly very good. The approximating function tends to 1 as r tends to R_{sup}.

Table 25.3 shows the values of *P* for the same values of r and R_{sup} as Table 25.2 does for μ^*.

We now have what we need: Table 25.1, columns 2 to 8 (under the heading 'My method based on $w(r)$ curves'), shows what we were looking for: the values of $\pi(r; R_{sup}) = \mu^* P$ which indicate the average probability that a switch in *C* (that is, at a certain value r of the rate of profit) is 'antineoclassical' for selected values of R_{sup}. This probability is higher the higher is r for a given R_{sup}; it is on the

Table 25.3 P, i.e. probability, according to my method based on the *w*(*r*) curves, that, if two techniques switch twice, the switch at the given value of *r* is associated with reverse capital deepening

r	R_{sup}						
	0.5	*1*	*2*	*5*	*10*	*100*	→∞
0.01	.1636	.1311	.1112	.0974	.0924	.0877	.0871
0.02	.2311	.1890	.1627	.1443	.1375	.1311	.1304
0.03	.2779	.2300	.1997	.1782	.1703	.1628	.1619
0.04	.3141	.2621	.2289	.2052	.1965	.1881	.1872
0.05	.3442	.2888	.2533	.2279	.2184	.2095	.2084
0.06	.3698	.3116	.2742	.2474	.2374	.2279	.2268
0.08	.4125	.3495	.3091	.2800	.2691	.2588	.2576
0.10	.4475	.3803	.3375	.3066	.2951	.2841	.2828
0.12	.4776	.4065	.3615	.3291	.3171	.3055	.3042
0.14	.5042	.4293	.3824	.3487	.3361	.3241	.3227
0.16	.5284	.4495	.4008	.3659	.3530	.3406	.3391
0.18	.5507	.4678	.4173	.3814	.3680	.3552	.3538
0.20	.5718	.4845	.4323	.3953	.3816	.3685	.3670
0.25	.6207	.5212	.4647	.4253	.4108	.3969	.3953
0.30	.6676	.5527	.4918	.4502	.4349	.4204	.4187
0.40	.7701	.6067	.5360	.4899	.4733	.4576	.4558
0.50		.6542	.5718	.5212	.5032	.4864	.4845
0.60		.6994	.6023	.5469	.5277	.5098	.5078
0.80		.7966	.6542	.5881	.5663	.5464	.5441
1.00			.6994	.6207	.5962	.5742	.5718
1.20			.7418	.6480	.6207	.5967	.5940
1.60			.8309	.6928	.6595	.6314	.6283
2.00				.7298	.6899	.6577	.6542
3.00				.8069	.7465	.7037	.6994
4.00				.8805	.7888	.7350	.7298
5.00					.8238	.7584	.7524
10						.8255	.8159
20						.8837	.8685
30						.9138	.8940

contrary lower the higher is R_{sup} for a given r. This shows that admitting no limit to R_{sup} tends to underestimate this probability relative to the – more plausible, it seems to me – cases in which the possible techniques can be presumed never to have an R_{sup} above a certain finite value. Anyway, the limits to which the probability tends for R_{sup} tending to $+\infty$ are also shown: the probability that a switch be 'antineoclassical' is about 10% for $r = 8\%$, about 13% for $r = 25\%$. Definitely not negligible.

In conclusion, D'Ippolito's 1987 results were deeply misleading. His probabilities are significantly lower, for the plausible values of the rate of profits, than the probabilities determined in the three alternative ways analysed here. In particular the calculation method which follows D'Ippolito's approach and statements most closely but corrects his logical slip (Table 25.1, next-to-last column: 'D'Ippolito corrected') arrives at probabilities enormously higher than his. The other two methods reach results less far from D'Ippolito's, but they nonetheless arrive at much higher probabilities than D'Ippolito's, especially for the low values of the rate of profits for which D'Ippolito obtains particularly small probabilities, e.g. for $r = 5\%$ they estimate probabilities around 8% against the 2.2% of D'Ippolito.

What also emerges is a significant dependence of the results on the assumptions about the distribution of the probabilities of the parameters chosen to characterize techniques. It is unclear how one might decrease the arbitrariness of these assumptions. Therefore, as stressed by Salvadori (2000), there is a danger that, by changing them, one may obtain nearly any result.[14] The significance and indeed the legitimacy of exercises such as the one attempted here (or the one by Mainwaring and Steedman 2000) are therefore doubtful. Still, if one believes this kind of exercise to yield some useful information, then the message appears to be that the Samuelson-Garegnani model supplies no basis at all for believing that the likelihood of 'antineoclassical' switches can be considered negligible – rather the opposite.[15]

Part III. A restrictive assumption in Mainwaring and Steedman and other studies

Leaving aside for the moment the doubts just raised about the relevance of the above type of exercise, a question implicitly posed by my results is why they are so different from the ones obtained by Mainwaring and Steedman (2000). They consider a two-sector model where both products are also capital goods and remain the same across changes of techniques, and try to estimate the '*a priori*' probability of reswitching between two techniques (differing in the sole production method of commodity 2) which switch at least once. They again propose to measure this probability as the ratio of two areas representing possible and equiprobable values of coefficients; this ratio depends on many parameters, so they have recourse to simulations. They find that if the two techniques switch at a given r^0 the probability (that they call $\psi(r^0)$) of a second switch depends on the technical coefficients, seldom going below 2%, often being in the range of 4% to 8%, and rising for certain combinations of values of the coefficients (combinations which, however, they deem of very low probability) above 50%. A series of curves showing how this probability varies with r^0, R, and various assumptions about the technical coefficients, suggests (at least to me; Mainwaring and Steedman do not explicitly propose a recapitulatory single estimate) an average probability $\psi(r)$ in the range 4% to 8% for plausible values of r. From this, they conclude that 'the probability of (frontier) re-switching at any two rates of profit in a restricted domain is very small – typically less than one per cent' (p. 345).

An immediate observation on this conclusion is that there is no reason why, in order to assess the plausibility of the supply-and-demand approach to value and distribution, one should be interested in the likelihood of *both* switchpoints of two reswitching techniques appearing on the frontier of the wage curves. This is an irrelevant issue once the really relevant question is admitted to be: if there is a switch on the frontier, what is the probability that it be an antineoclassical one? This is the question relevant for the possibility of antineoclassical changes in technique as distribution changes, and is indeed the question asked by D'Ippolito, and by myself in Parts I and II. Therefore Mainwaring and Steedman should have rather concentrated on estimating the probability that if two techniques switch, they not only reswitch but do so at a *lower* rate of profit,[16] neglecting the question whether either switch is dominated, or whether both switches will be inside a 'restricted domain'. This probability is certainly lower than the average $\psi(r)$, which also considers second switches at a *higher* rate of profit, but if one can assume as an indication the values I obtain for the Samuelson-Garegnani model for the probability P that, when two techniques that switch also reswitch, the other switch be at a lower rate of profit (cf. Table 25.3), then for not very high values of R[17] one easily obtains that $\psi(r)$ should be no more than halved. So I would provisionally guess for the Mainwaring-Steedman model an average probability that a frontier switch be antineoclassical around 2 to 4 per cent instead of less than 1 per cent.

Now an *average* probability of antineoclassical switches around 2 to 4 per cent (which means that in concrete instances their frequency could be even considerably higher) appears sufficient to argue the 'potential generality' (Garegnani 1990, p. 72) of very antineoclassical behaviours of the value-capital/labour ratio. Still, the question remains why one obtains so much lower probabilities than for the Samuelson-Garegnani model. Further research on this issue would be necessary, but in all likelihood a main reason is that, as noted by Ciccone (1996, p. 45, fn. 8), the model assumes both goods to be common to both alternative techniques, so techniques that switch differ in the productive method of only one sector. On the contrary, in real economies it very seldom happens that different methods of production of a commodity do not require different intermediate goods or machines – so the cases in which at a switchpoint the capital goods directly and indirectly utilized in the production of the numéraire do not change should be considered exceptional. This is accepted, indeed made central, in the Samuelson-Garegnani model where a change of techniques always changes both methods because the capital good changes; it is on the contrary excluded in the Mainwaring-Steedman model, and the probability of reswitching is certainly reduced by this restrictive aspect of the model, because then reswitching can happen only if both $w(r)$ curves are concave or both are convex. Some indication that the reduction is very great can, I think, be obtained from a comparison with the Samuelson-Garegnani model, where this constraint on the shape of the $w(r)$ curves of reswitching techniques does not hold. For the latter model I obtain *very* high values (cf. Table 25.2) for the probability μ^* that two techniques which switch also have a second switch, for example $\mu^* = 37\%$ for $r = 10\%$ (against values of $\psi(r)$ which seldom go above 4% for $r = 10\%$ in the Mainwaring-Steedman model, cf. Figures 3 to 6 of their paper). True, in the Samuelson-Garegnani model there is only one capital good in each technique, and admitting two capital goods, both inputs to each other, which can both change with a change of technique, would require considering a three-goods model (at least one good, the consumption-numéraire good, cannot change across techniques); but I can see no reason why this modification should significantly restrict the shapes that $w(r)$ curves of reswitching techniques can have, and therefore should significantly reduce the likelihood of double switching relative to the Samuelson-Garegnani model. Therefore I consider my μ^* values more credible than Mainwaring and Steedman's $\psi(r)$.

The assumption that at switchpoints on the frontier the two techniques differ in the method of only one of the industries relevant for the determination of the $w(r)$ curve is also made by D'Ippolito (1989) and by Han and Schefold (2006). The effect of this restrictive assumption on the likelihood of antineoclassical switches in models with many industries awaits study. If changes of technique change the capital goods utilized, more than one method (of the ones affecting the shape of the $w(r)$ curve[18]) changes at a switchpoint. Accordingly, the set of possible shapes of $w(r)$ curves passing through a given (r, w) point is considerably enlarged. Thus consider the model used by D'Ippolito (1989). He assumes a given (A_1, l_1) technique for an n-goods economy, which generates a certain $w(r)$ curve. He fixes $r = r^*$ and chooses units for the several commodities such that, having fixed the numéraire, $w(r^*) = 1$ and $p = (1, \ldots, 1)$. Then by Montecarlo methods[19] he randomly chooses different coefficients for one industry, say the numéraire industry 1, such that the new technique (A_2, l_2) (that differs from the first one only for the method of the first industry) switches with the old one at r*, that is, yields the same $w = 1$ and the same $p = (1, \ldots, 1)$ for $r = r^*$; this means that, with a_{1j}, l_1 respectively the technical coefficient of input j and of labour in industry 1, the new coefficients of industry 1 must satisfy

$$(a_{11} + a_{12} + \ldots + a_{1n})(1 + r^*) + l_1 = 1.$$

If n is great, if all industries affect the shape of the $w(r)$ curve, and if 'no sector of the economy is 'large' ' (Schefold 1997a, p. 279); that is, if changes in the technical coefficients of only one sector cannot greatly affect the $w(r)$ curve), the new $w(r)$ curve will be very close to the old one: for example the possible variation of the coefficients of industry 1 will not be able significantly to modify the maximum wage rate or the maximum profit rate. If on the contrary the new technique uses different *quasi-basic* goods (that is, goods entering the numéraire basket or its direct or indirect means of production), then the number of quasi-basic industries that survive with an unchanged method at a switchpoint can be any number from $n - 2$ to zero, and prices at r^* need remain equal to 1 only for the commodities which are quasi-basic in both techniques, possibly only one commodity;[20] the new $w(r)$ curve can differ from the old one much more drastically. For example it can have inflection points in cases in which these would be excluded by D'Ippolito's conditions. The set of possible shapes of reswitching $w(r)$ curves is then enlarged as well.

It may be countered that this enlargement of the set of possible shapes of switching and of reswitching $w(r)$ curves does not necessarily mean that reswitching becomes relatively more probable; however, the effect would seem to be precisely this one when contrasting the Mainwaring-Steedman model with the Samuelson-Garegnani model, so it seems legitimate to suspect that this might well be the general effect.

Part IV. On the relevance of the probability of reswitching

But why is the probability of occurrence of reswitching considered important? As argued by many authors (e.g. by myself in Petri 2004, ch. 6), the simple *possibility* of reswitching[21] suffices to destroy the legitimacy of the conception of capital as a single factor of production of variable 'form'; no possibility is thereby left of basing a general approach to value and distribution on that conception. This is particularly clear for the endowment of capital: the endowment of a non-existent factor does not exist; no possibility is thereby left of writing acceptable general equilibrium models with a given endowment of a single factor 'capital' when capital goods are in fact heterogeneous. But the problem with specifying the *endowment* of capital as a single quantity for economies with heterogenous capital goods had been perceived by the 1930s and was the main reason behind the shift from long-period general equilibrium models relying on capital as the value factor to the modern, neo-Walrasian versions. However, this shift was not accompanied by doubts about the working of capital-labour substitution in response to changes in the rate of interest (Petri 2004, pp. 156 – 60); as a result, the thesis remained dominant to this day of a negative elasticity of investment to the rate of interest. This characteristic of the investment function is fundamental for neoclassical macroeconomics, international economics, growth theory, etc. because it is necessary in order to argue that the rate of interest is capable of bringing investment into equality with full-employment savings, and thus to give some plausibility to the tendency toward a full-employment equilibrium which is fundamental for the neoclassical approach.[22] Nowadays there are several attempts to derive this negative elasticity without relying on capital-labour substitution, but they are all vitiated by grave deficiencies (Petri 2004, ch. 7); therefore if it could be concluded that it is impossible to rely on the traditional neoclassical conception of capital-labour substitution, the negative elasticity of investment with respect to the rate of interest would lose all *theoretical* credibility and therefore *all* credibility (it is generally admitted that it has very little *empirical* support), and then only very ignorant or very dogmatic economists could remain neoclassical.

Now, reverse capital deepening derived from reswitching has been used by the critics precisely to refute the neoclassical conception of capital-labour substitution and thus to criticize the traditional investment function. I view the insistence on a low probability of reswitching as an attempt to counter this criticism of the interest-elastic investment function.

The implicit argument (I am not aware of it having been clearly spelled out in its entirety) would appear to be the following.[23] If reverse capital deepening is highly improbable then, as Schefold (2010, p. 122) has put it: 'each single small change of methods of production in different industries can only exert a small effect on the aggregates, and if the system is large and the changes are many, rare paradoxical [i.e. antineoclassical, F. P.] changes will, as it were, disappear in the noise of frequent transitions'; in other words a strong predominance of 'neoclassical' changes will imply a value-capital/labour ratio and hence an investment function almost everywhere negatively elastic with respect to the rate of profit (rate of interest), such that the portions where that is not the case will not be able to determine significant indeterminacies of the equilibrium interest rate on the investment-savings market. The idea seems to be representable as in Figure 25.5a, where the relatively small backward portions of the capital demand curve (and hence of the investment function derived from it – see below for a discussion of how) appear incapable of causing significant indeterminacies of the level of the rate of interest that ensures equilibrium between full-employment savings and investment, so that one can speak of stability of the savings-investment market (as long at least as the savings function is not backward-bending, as one should assume if one wants to leave aside problems due to income effects in consumer choices).

A first problem with this argument, that emerges from the considerations advanced in Parts I to III, is that the existing studies on the probability of

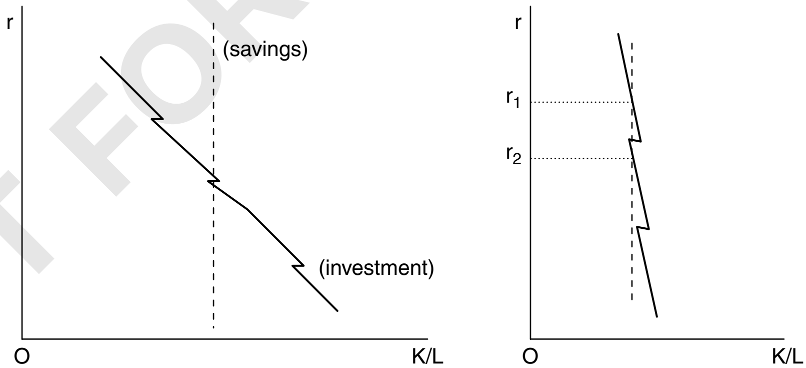

Figure 25.5 The curves can be interpreted both as demand-for-capital and capital-supply, or as investment demand and flow-supply of savings (in which case the *L* at the denominator on the abscissa is the flow of labour 'freed' by the gradual closure of the older plants).

reswitching do not support a sufficiently low probability of reswitching. Even conceding some meaningfulness to these studies (against the doubts raised above), as already declared, even as low an average probability of antineoclassical switches as on the frontier of 2 to 4 per cent – and I have pointed out reasons to consider these estimates too low, possibly vastly too low – would not bar the possibility, with nonnegligible probability, of instances in which the percentage of antineoclassical switches on the frontier were higher, even considerably higher, and as a result the demand-for-capital curve were utterly unable to support the thesis of a unique and stable equilibrium of the savings-investment market. Some historical realization of such instances should be expected to have happened, as I have observed elsewhere;[24] and market economies have been able to function all the same, which strongly suggests that the forces determining distribution and employment are not the ones postulated by the neoclassical approach.[25]

But the argument has other weaknesses too, and I proceed to point them out, although briefly for space reasons.

A second problem with the argument is that it requires that *reverse capital deepening*, rather than reswitching, be highly improbable. Now, reverse capital deepening can also be due to 'price Wicksell effects'. Consider a single everywhere strictly concave $w(r)$ curve: with no change in physical quantities, the value of capital per unit of labour in terms of the net product increases with the rate of profit. The same positive correlation between rate of profit and capital-labour ratio can be observed even for intervals of values of r where the $w(r)$ curve is convex, if it is concave in preceding intervals. The likelihood that a $w(r)$ curve be concave or have concave sections would appear to be not less than that it be convex or with convex sections. A series of concave $w(r)$ curves succeeding one another on the frontier may cause the value of capital per unit of labour to have many upward-sloping sections, or even to be entirely upward-sloping even in the absence of reswitching as demonstrated by Garegnani (1970) under an assumption of continuous frontier switching, i.e. infinite alternative techniques. Figure 25.6 provides a graphical example for a case with a finite number of techniques. In this example (which is compatible with the assumptions of D'Ippolito's model as well as with more complex models) the absence of reswitching guarantees that the change of the value of capital at switch points is not 'antineoclassical', nonetheless the overall behaviour of the function connecting the value of capital per unit of labour to the rate of interest is far from the one normally assumed in neoclassical analyses.

The question thereby raised is whether reverse capital deepening caused by positive price Wicksell effects can be an additional cause of problems for neoclassical theory, besides the reverse capital deepening caused by reswitching. The answer is yes. Savings are generally admitted to depend not only on income but also on wealth. Therefore the supply of gross savings will be affected by price Wicksell effects too: in a given situation of capital stock adjusted to long-period technical choices and income distribution, and equilibrium between investment and savings, for small variations of r the change in the supply of savings can be decomposed into two, the effect of the change in the rate of interest (rate of profit)

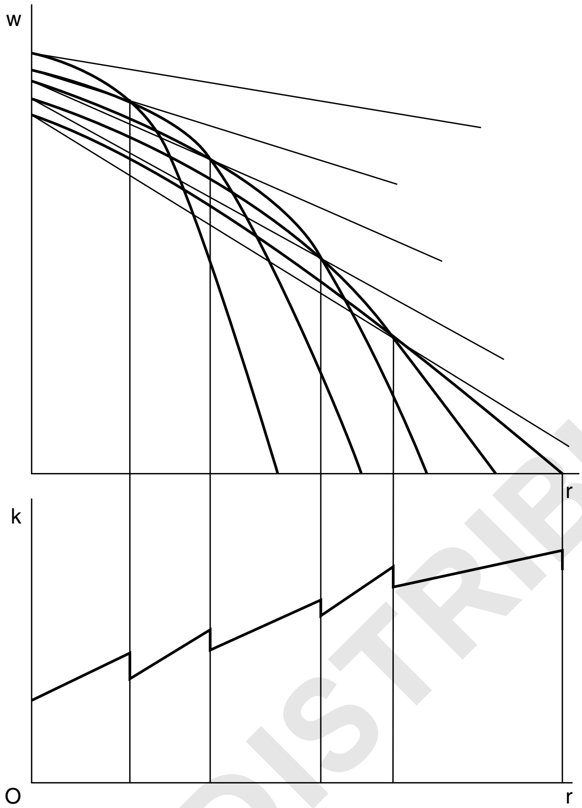

Figure 25.6

with an unchanged wealth of the owners of capital goods, and the effect of the change in wealth due to the change in the value of capital goods, with an unchanged rate of interest. If the second effect coincided with the price Wicksell effect upon the demand for capital, and assuming no 'perverse' income effects (no negative effect of changes in the rate of interest on the propensity to save), stability could be endangered only by antineoclassical switches of technique (Fratini 2009). However, there is no guarantee that a change in the value of capital goods with an unchanged rate of interest will induce an equal change in gross savings. Thus suppose that a rise in *r* from a situation of equality between investment and savings does not cause switches to a different technique nor changes in the amounts produced but raises the value of the existing vector of capital goods (which has not changed) by 5% because of price Wicksell effects and that as a result investment (in value) increases by 5%; there is no guarantee that savings will, because of the rise of *r* and of the wealth of consumers, rise by 5%; and if savings rise less than 5%, the equilibrium between savings and investment is unstable. Thus price Wicksell effects are an additional possible cause of problems for the stability of the savings-investment market in the neoclassical approach.

A third problem with the argument is that it requires that the 'neoclassical' switches be able to guarantee a not only essentially decreasing, but also *sufficiently elastic* demand-for-value-capital function: as always with equilibria, a low elasticity of the demand function risks determining an implausible equilibrium value of the variable under discussion (e.g. a negative rate of interest, or so high a rate of interest and hence of profit that the corresponding real wage is below subsistence) as well as implausible comparative statics (small shifts of the savings function or of the labour supply function, or of the investment function owing to technical progress, would be liable to cause enormous jumps of income distribution). Also, the lower the elasticity of the investment function ensured by 'neoclassical' switches, the more dangerous the occurrence of antineoclassical switches because these, even if infrequent and with modest effects on the demand for capital because influencing only a small part of the economy, decrease anyway somewhat the overall elasticity of the function, with effects on the extent of possible indeterminacies of the equilibrium rate of interest potentially the greater, the lower this elasticity, cf. Figure 25.5b where, differently from Figure 25.5a, the distance between the two locally stable equilibrium rates of interest is *not* negligible.

Unfortunately none of the authors who have attempted to determine the probability of antineoclassical switches for economies with many industries has gone on to analyse the elasticity of the value of capital per unit of labour with respect to the rate of profit (rate of interest) in the models they examine. But there are reasons to suspect a very low elasticity even apart from antineoclassical switches. The empirical enquiries of Ochoa (1989), Petrović (1991), Tsoufildis and Maniatis (2002) and others[26] conclude that $w(r)$ curves are nearly linear; Schefold (2010, p. 127) concurs, on the basis of the results of the empirical enquiry of Han and Schefold (2006); Bidard and Schatteman (2001) bring some analytical support to this view (cf. Schefold 2010, p. 122). This suggests that price Wicksell effects are not very relevant in the aggregate for observed techniques. This decreases the relevance of the problem discussed above but raises a new problem. Han and Schefold (2006) find a very small number of switchpoints, ten on average, on the envelopes of the techniques with 33 different goods that they derive from empirical input – output tables; this result, if generalizable beyond the inevitable aggregation and not very great income variation connected with the use of input – output tables from similar advanced countries, means that a majority of industries experiences no change at all in optimal production methods as distribution changes within a realistic range.[27] If then one accepts Schefold's argument that generally a single switch in a many-goods economy will only have a very modest effect on the $w(r)$ curve, the implication is that changes in technique induced by changes in distribution change the value-capital/labour ratio very little. On the other hand, if the near linearity of $w(r)$ curves implies (as argued in the same papers) that the changes in relative prices induced by changes in distribution are generally small, substitution due to consumer choice (even assuming it is not 'perverse' owing to income effects) will be weak too. But then the investment function derived from the demand-for-capital function will be nearly vertical. The neoclassical approach is undermined anyway.

A fourth problem with the argument concerns the assumptions about the level of labour employment behind the derivation of the investment function.

The level of the rate of profit (rate of interest) determines the desired value-capital/labour *ratio*, which becomes a demand-for-value-capital function when one assumes a given (i.e. full) employment of labour.[28] The concrete role of this function is to allow the derivation of the *flow* of gross investment implicit in the demand for the *stock* of capital, owing to the need to re-equip with the optimal capital goods the employed labour as capital goods are gradually used up; leaving aside for simplicity investment in circulating (intermediate) capital goods (anyway only the *variations* of their amounts are considered in current national income accounts), there results what in Petri (2004 p. 127) I have called the *long-period investment function*: at each rate of profit it measures the value of capital to be employed in new plants at the corresponding optimal value-capital/labour ratio, assuming normal (long-period) prices and a given flow of labour to be employed in new plants (because 'freed' by the gradual closure of existing plants as they reach the end of their economic life, in a situation of full labour employment and normal utilization of existing plants on average).[29] This function, a reduced-scale copy of the demand-for-value-capital function,[30] could be argued to give a good approximation to the average value of investment in durable capital over sufficiently long periods (if capacity utilization was normal on average), even if most durable plants during that period were still *not* adapted to the new level of the interest rate because of even longer durability and were only earning residual quasi-rents: prices would anyway be determined by the newer plants, better adapted to current income distribution; the irregularities of the 'freeing' of labour owing to irregular age distribution of durable capital could be argued not to alter the fundamental terms of the question if one was interested in average investment over sufficiently long time periods.

But the long-period investment function needs the full employment of labour. Essentially, it reflects the capital/labour ratio *in new plants*, so investment is determined only if the denominator of the ratio is given. With unemployment, a given capital/labour ratio in new plants leaves investment indeterminate: the flow of labour employed in new plants can be greater than the flow of labour 'freed' by the closure of the oldest plants, with a resulting gradual reduction of unemployment; or it can be smaller, with a resulting gradual increase of unemployment. Nowadays this is often forgotten, and one frequently meets textbooks where the interest-elastic investment function used in the IS-LM model is derived from a decreasing marginal product of capital, forgetting that – even conceding the validity of the notion – a decreasing marginal product of capital requires a given employment of labour, which is *not* assumed in IS-LM analysis. If what is at issue is the validity of the neoclassical approach, the full employment of labour cannot be *assumed*. In fact, empirical evidence suggests the normal presence in capitalist economies of unemployment and of a capacity utilization vastly inferior to the technically maximum one; furthermore, the supply of labour hours can be varied even without changes in the number of employed labourers by varying the use of part time and overtime (over longer periods there is immigration and changes in the rate of participation that render labour supply largely determined by labour demand). The resulting considerable elasticity of production inevitably attributes a relevant role to aggregate demand in the determination of aggregate output; the resulting utilization of productive capacity will then be a main force affecting desired variations of productive capacity and hence investment.

There has been a neoclassical approach to investment – now out of fashion – that tried to take account of this issue: the one of Jorgenson (1963) then popularized in the macroeconomics textbook of Dornbusch and Fischer (e.g. the 1987 edition). In this approach the rate of interest determines the desired K/L ratio but the denominator in this fraction is not determined by a full employment assumption: rather, industries determine their desired capital stock on the basis of the expected levels of demand. At the aggregate level, the influence of the rate of interest on investment is then obtained as follows: the rate of interest selects the capital-labour proportion on the aggregate isoquant corresponding to the planned level and composition of aggregate output; the desired capital stock changes if either the rate of interest, or planned output (i.e. expected demand), or both, change; net investment is effected to adjust the actual capital stock to the desired capital stock. How the speed of adjustment is determined is left somewhat vague by Dornbusch and Fischer and is only econometrically estimated by Jorgenson; but what interests us now is that with this approach: a) for a given level of aggregate demand, the elasticity of the desired capital stock (and hence of investment[31]) to the rate of interest is less – possibly considerably less – than if the denominator of the *K*/*L* fraction were given: this is shown in Figure 25.7, where a change in distribution that changes the optimal *K*/*L* ratio from α to β causes an increase of desired capital from K_1 to K_3 if labour employment is fixed at L_1, from K_1 to K_2 if output is fixed; *b*) investment relevantly depends on the variations of aggregate demand, i.e. on the acceleration principle.[32]

Thus the result of taking into account the presence of unemployment into an otherwise neoclassical approach to capital and investment is to have to admit possible significant multiplier-accelerator interactions, and to decrease the interest-elasticity of the investment function derived from capital-labour substitution.

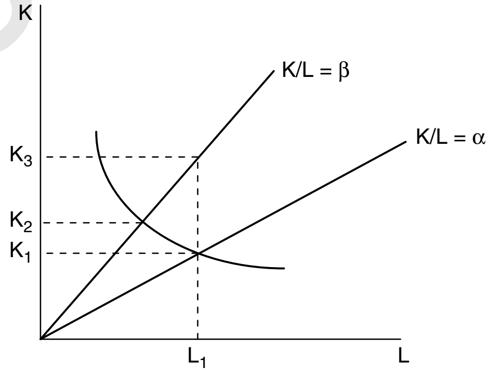

Figure 25.7

Whatever (low) interest-elasticity of investment might be presumed probably to exist when account is taken of this last observation and of the arguments of the preceding sections will then be in all likelihood swamped by the greater influence of aggregate demand and its variations – a conclusion certainly not contradicted by the available empirical evidence.[33]

It seems clear then that the anti-Keynesian argument of a spontaneous tendency toward full employment if money wages are flexible, based on the 'Keynes effect', is indefensible. There appears to be no reason to believe that decreases in money wages will have a stronger positive effect on investment via the (highly doubtful anyway) effect on and through the rate of interest, than a negative effect on investment due to the – initial, at least – decrease in real wages[34] and hence in consumption expenditure. The consequent persistence of unemployment[35] and the elasticity of production imply very different policy implications from the currently dominant ones. For example, the flexibility of production in response to changes in demand implies that there is no necessary influence, in the short as well as in the long period, of changes in real wages on the demand for labour. In existing plants, where capital already has a given 'form', higher real wages will bring about little or no change in output per unit of labour: employment will depend on capacity utilization which will depend on aggregate demand. In new plants, the flexibility of production of capital goods industries will generally pose no problem with obtaining the inputs required by the adoption of the new most profitable methods of production on the scale suggested by the expected level of aggregate demand, even if the latter is increasing considerably. Thus (apart from political reactions) there generally is no incompatibility between more employment and higher wages; all that is required is that the higher wages be accompanied by a stimulus to aggregate demand. This will be so even when it were the case that a higher wage implied a shift to more value-capital-intensive techniques and therefore required more savings: the increase in savings will be brought about by the increase in aggregate output.[36]

It can be concluded that the argument that the probability of reswitching is very low, disputable as it is, anyway would not rehabilitate traditional neoclassical analyses.

Appendix A

I take from D'Ippolito (1987, p. 32) the following determination of the surface $F(r, v)$. Refer to Figure 25.8, which is in fact the same as Figure 25.2a. $F(r, v)$ is the difference between the areas of the triangles ABC and $A'B'C$. Since:

$$OC = BC = 1/\rho$$

$$OM = ME = 1$$

$$CM = r/\rho$$

$$AC/CB = A'C/CB' = A'M/ME = A'M = v$$

$$A'C = A'M - CM = v - r/\rho$$

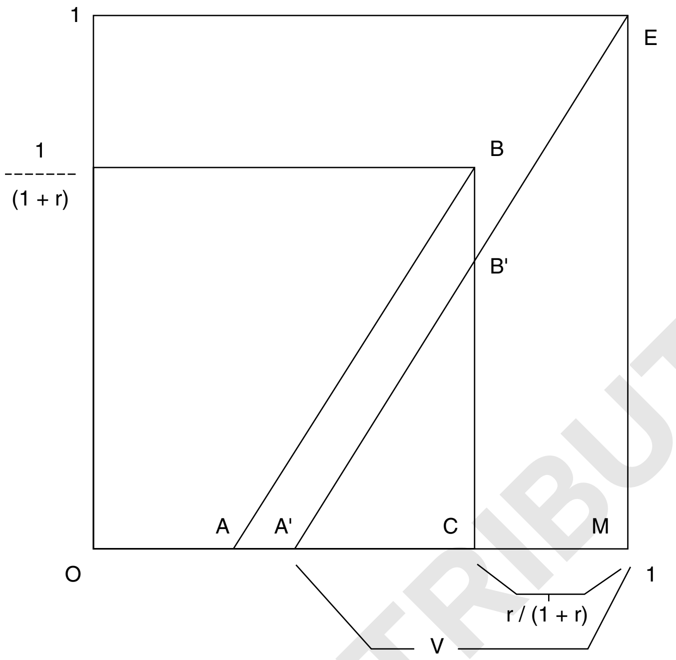

Figure 25.8

one obtains the areas of the two triangles as, respectively,

$ABC = (AC \times CB)/2 = CB^2 v/2 = v/(2\rho)$

$A'B'C = (A'C \times CB')/2 = A'C^2/(2v) = (v - r/\rho)^2/(2v)$ as long as $v \geq r/\rho$,

otherwise $A'B'C = 0$.
Hence the area of F is

$$F = \frac{1}{2\rho^2}[v \frac{1}{\rho}(\rho v - r)2] \text{ if } v \geq r/\rho;$$

$$F = v/(2\rho) \text{ if } v < r/\rho.$$

$$(v - r/\rho)^2$$

Hence $Z(r, v) = F(r, v)/D(r, v) = F(r, v)/ABC = 1 - \frac{(v - r/\rho)^2}{\rho^2}$ if $v \geq r/\rho$, otherwise $Z(r, v) = 1$. Therefore

$$Z^*(r) = \text{r}/\rho + \int_{r/\rho}^{1} [1 - (v - r/\rho)2/\rho 2]dv.$$

Notes

1 This is admitted by several neoclassical economists. Two examples: 'Does it not take time to establish equilibrium? By the time equilibrium would be established will we not have moved on to another "week" with new conditions, new expectations, etc.?' (Bliss, 1975, p. 210). 'In a real economy, however, trading, as well as production and consumption, goes on out of equilibrium. It follows that, in the course of convergence to equilibrium (assuming that occurs), endowments change. In turn this changes the set of equilibria. Put more succinctly, the set of equilibria is path dependent [. . .] [This path dependence] makes the calculation of equilibria corresponding to the initial state of the system essentially irrelevant' (Fisher, 1983, p. 14); note how the reference to production implies that Fisher is referring not only to possible exchanges of endowments among consumers, a problem of secondary importance, but also to changes in the total endowment of each produced means of production.

2 That the definition of a state of rest of prices should include the full employment of labour becomes then highly questionable. The reason behind such an inclusion in the marginalist approach is the presumption that unless labour demand equals labour supply, the real wage will change. But this presumption is difficult to justify unless one can argue that the change in real wage needed to reach equality between labour demand and labour supply is not drastic. This requires both a downward-sloping and sufficiently elastic demand for labour, and some mechanism ensuring that if there is unemployment and wages decrease and this induces firms to hire more labour, the increased output will be sold without problems, that is, aggregate demand will increase in step with aggregate output (Say's Law). Neither assumption is supported by modern GET owing both to income effects, and to the four difficulties of GET which render the theory silent on the behaviour of economies where adjustments are time-consuming. Whether wages will change in the presence of unemployment becomes then an open question: unless wage reductions considerably increase the demand for labour, a downward 'stickiness' of wages becomes both *a necessity* in order to avoid absurd conclusions such as wages falling to zero (Petri 2004, pp. 319–20), and *a plausible outcome* of social interactions on the basis of historical experience – an outcome that classical authors, for example, considered obvious. The assumption of indefinite wage decreases as long as there is unemployment could be considered by marginalist economists a natural premise to the definition of equilibrium only because their theory argued with some apparent plausibility that full employment could be reached by *plausible* wage decreases. Already with Keynes the problems with such an argument prompted the admission of social mechanisms rendering (money) wages 'sticky'.

3 Representative examples are Hicks (1965, p. 156; 1973, p. 44); Eltis (1973, Ch. 5); Malinvaud (1986). For contrary views cf. e.g. Garegnani (1990, pp. 71–2); Ciccone (1996).

4 The widespread acceptance of the term 'perverse' to characterize phenomena that are simply in contradiction with the predictions of one's preferred theory appears to betray a quasi-religious outrage at the emergence of phenomena questioning neoclassical certitudes, which has little to do with a correct scientific attitude. It is difficult to imagine a physicist calling 'perverse' the results of experiments contradicting accepted theories: in the natural sciences the reaction would more probably be one of excitement because unsuspected new aspects of reality would be emerging that could be expected, once understood, to permit a better mastery of the world.

5 'Not less than', because the probability depends negatively on the assumed upper limit R_{sup} of the maximum rates of profit of the alternative techniques, and the values given in the text are the ones for $R_{sup} = +\infty$.

6 The vertical intercept of the $w(r)$ curve measures the value of net output per unit of labour, i.e., in our case, the physical production of consumption good per unit of total

labour employment (the economy is assumed to be stationary). Let y_1, y_2 be these net outputs per unit of labour for technique 1 and 2, and assume $y_1 < y_2$. Then labour employment *per unit of output*, $L = 1/y$, is smaller with technique 2. Put net output equal to 1; since net output must equal net income i.e. $y = 1 = wL + rK$, if L is smaller in technique 2, then K must be greater. But the value of capital per unit of labour will also be greater at a switchpoint if the vertical intercept is greater, cf. equation (25.6).

7 The meaning of $v < 1$ is that the capital per unit of product in the sole consumption good industry must be greater with technique 2 than with technique 1.

8 It might on the contrary be argued that very low values of the coefficient a, implying very high values of the maximum rate of profits R, are less and less plausible the more they approach zero. It might also be argued that a cannot be very close to $1/(1 + r)$.

9 He appears here to mean all values of v between 0 and 1, as made clear by the limits of integration in footnote 15, p. 18 of his article; if one were to interpret him literally then, since v can vary from 0 to $+\infty$, the probability that v will fall in any finite interval would be zero, i.e. the probability would be all concentrated at the value $v = +\infty$. The *a priori* symmetry of the possibilities $v < 1$ and $v > 1$ suggests instead to consider the two cases as equally probable for a random picking out of two techniques (giving or not rise to a switch: cf. section 3), i.e. to consider the probability that $v' < v < v' < 1$ with v' and v' assigned, equal to the probability that $1/v' < 1/v < 1/v'$. D'Ippolito appears to concur in this view (cf. below in the text).

10 I am unable to accept Ciccone's attempt (1996, pp. 51–4) to justify D'Ippolito's procedure. Ciccone writes (p. 52, my translation): 'Because of the symmetry between the conditions $v < 1$ and $v > 1$, the $P_{me}(r)$ calculated for values of v included between 0 and 1 comes out to be in fact equal to the average probability obtainable for values of v included between 1 and $+\infty$'; but this is false, because if one follows D'Ippolito in calling technique 2 the one dominant to the right of r, then the second average probability is simply zero.

11 It may be noticed that if one associated to each $v < 1$ the corresponding $1/v$, the eligible portions of OCBQ would sum to exactly the area of OCBQ.

12 To consider all points in OCBQ equally probable is clearly arbitrary in that they would not be equally probable if one decided e.g. that it is all admissible couples (α_1, α_2) that are equally probable. This arbitrariness is ineliminable from exercises of this kind.

13 Cf. note 8 above on the need to replace v with $1/v$ when $v > 1$ in order to avoid having a zero probability of all finite intervals of values of v.

14 Salvadori stresses in particular that a technique can be characterized in many different ways, each one based on different parameters, so that an assumption – arbitrary as it is anyway – of 'equal probability' of all values of these parameters within their acceptable range will generate different results for different characterizations of techniques. Indeed had I not replaced R with a in section 5, the numerical results would have been radically different.

15 Laing (1991) attempts a similar comparison of areas, for an 'Austrian' model where the consumption good is produced by current labour, labour employed one period earlier, and labour employed two periods earlier; all he is able to argue is that 'there is a much bigger volume [of possible coefficient values] where double-switching does not occur than where it does' (Laing 1991, p. 187); how much bigger, he is unable to estimate numerically; the sole case in which he obtains a numerical estimate is the case which he deems the most favourable possible to RCD; the ratio of the two 'volumes' in that case is 2:1, indicating a 33% probability of RCD, which makes it likely that the range of probabilities for this model must be of the same far-from-negligible order of magnitudes as the ones found above for the Samuelson-Garegnani model. Laing's conclusion that RCD and reswitching 'are a possibility but are exceptional' (p. 185) appears therefore totally unwarranted.

16 Mainwaring and Steedman prefer to speak of 'double equi-profitability' to mean that two $w(r)$ curves cross twice, and, differently from the terminology adopted here, to restrict the term 'reswitching' to the cases of double equi-profitability where both switches are on the frontier.

17 From their Figures it would seem that Mainwaring and Steedman prefer to assume that R is not greater than 1.

18 It is always possible to 'border' each technique matrix with industries producing, as non-basics, the capital goods only utilized in other techniques; then all techniques produce the same goods and one obtains that at a switchpoint the method of only one industry changes; but what is relevant for the change of the $w(r)$ curve is whether there is a change of the goods (that can be called *quasi-basic* or also *wage goods*) entering the numéraire basket (but these cannot change) or directly or indirectly used in its production.

19 The chapter does not specify the procedure adopted to generate the random coefficients, and in particular the probability distribution assigned to them including the probability that a coefficient be zero; an announced longer paper explaining these issues has not been published owing to Professor D'Ippolito's death; thus a thorough assessment of the plausibility of his 1989 results appears impossible.

20 The fact that D'Ippolito (1989) assumes that all prices remain equal to 1, that is, that all prices are the same at a switchpoint, shows that he is assuming that all quasi-basic goods are common to both techniques. The same assumption is stated by Han and Schefold (2006) on p. 741.

21 Actually, the simple possibility of the phenomenon that makes reswitching possible: the inversion of the movement of relative prices as the rate of profit rises, cf Sraffa 1960, p. 84, and Petri 2004, pp. 210–16.

22 On the Pigou or real-balance effect cf. Petri 2004 pp. 291–4; its weakness is admitted even by Patinkin.

23 In discussing this argument, I shall leave aside the arbitrariness in determining the value-capital/labour ratio due to the arbitrariness in the choice of numéraire. Potestio (2010) has used this arbitrariness as a criticism of some presentations of the Sraffian critique of neoclassical capital theory, but the point of those presentations was that *even* leaving aside this problem as in certain cases it is possible to do (for example, by assuming a single consumption good which is then the natural numéraire because appropriate to measure the sacrifice – the potential consumption given up – connected with acts of saving) still powerful criticisms are possible. If the purpose is criticism, it is admissible to concede some ultimately illegitimate aspects to the argument to be criticized if that allows highlighting better the weakness of a more fundamental assumption.

24 'We have now had two centuries of capitalism, with all its technical changes and national peculiarities, so we have had very many "random" extractions of sets of alternative techniques. Thus even very low probabilities of reverse capital deepening would not make it unlikely that, at least in some countries and some historical periods, the capital-labour ratio schedule had upward-sloping sections, which should have resulted in at least some cases in phenomena that, to the contrary, have not been observed' (Petri 2004, p. 254).

25 'However small the evaluated probability of the instances in which the principle of substitution does not operate, obviously prices and incomes would take shape, and would therefore have an explanation, also in those circumstances. One would thus be implicitly admitting the existence of a theory of distribution, alternative to the neoclassical one, and without any basis for excluding that this alternative theory, differently from the neoclassical one, may apply to the generality of cases' (Ciccone 1996, p. 42, my translation).

26 These can be obtained from the references in Tsoufildis and Maniatis (2002).

27 Casual observation appears to suggest very little change in technology when firms move production plants from high-wage to low-wage countries, and this appears to

confirm a predominant independence of modern optimal production methods from (realistic) changes in wages.

28 Or a fully employed supply of labour which is a non-decreasing function of the real wage.

29 If the rate of interest changes and normal prices with it, it is only in new plants that the new optimal production methods can be adopted. Already existing plants and other durable capital goods will go on being utilized as long as they earn non-negative quasi-rents, and there is little reason to assume that normal labour utilization in them will be relevantly affected by a higher real wage, given the little room for changes in production methods once the fixed plant is built. Thus, labour will be combined with capital goods adapted to new optimal technical choices only gradually, as existing plants gradually reach the end of their economic life and are replaced by new plants.

30 Cf. Garegnani (1978–79, pp. 35, 64–5).

31 The convenience of keeping existing plants in operation as long as quasi-rents are non-negative still holds, so if output does not change investment will be again an opportune reduced-scale copy of the demand for capital.

32 Dornbusch and Fischer do not place great stress on this implication of their approach, and do not recognize that it seriously questions the central role of the IS-LM model in their textbook: the relevant influence of *variations* of output on investment makes the IS curve a construction of little significance.

33 Cf. Petri 2004, pp. 257–8. It is an interesting mental exercise to try to imagine the effect of teaching investment theory starting directly from the available empirical evidence, without any previous indoctrination with neoclassical notions of capital-labour substitution.

34 Some time will be required for prices to decrease by the same percentage as money wages, even assuming such a price decrease to happen; and during this time real wages are reduced.

35 This unemployment must be considered involuntary, because it does not depend on workers refusing real wage reductions: these would not get them a job anyway.

36 Thus one might say, in neoclassical language, that owing to the adaptability of production to demand, relative factor proportions adapt to income distribution rather than the other way round.

References

Bidard, C., and T. Schatteman, 2001, ‘The Spectrum of Random Matrices’, *Economic Systems Research*, 13: 3, 289–98.

Bliss, C. J., 1975, ‘The reappraisal of Keynesian economics. An appraisal’, in M. Parkin, A. R. Nobay, eds, *Current Economic Problems*, Cambridge: Cambridge University Press.

Bloise, G., and P. Reichlin, 2009, ‘An Obtrusive Remark on Capital and Comparative Statics’, *Metroeconomica* 60: 1, 54–76.

Ciccone, R., 1996, ‘Possibilità e probabilità di comportamento “perverso” del capitale’, *Studi Economici* 58: 1, 41–73.

D’Ippolito, G., 1987, ‘Probabilità di perverso comportamento del capitale al variare del saggio di profitto. Il modello embrionale a due settori’, *Note Economiche*, 2, 5–37.

D’Ippolito, G., 1989, ‘Delimitazione dell’area dei casi di comportamento perverso del capitale in un punto di mutamento della tecnica’, in Luigi Pasinetti, ed., *Aspetti controversi della teoria del valore*, Bologna: Il Mulino, 191–8.

Dornbusch, R. and S. Fischer, 1987, *Macroeconomics*, 4th edn, New York: McGraw-Hill.

Eltis, W., 1973, *Growth and Distribution*, London: Macmillan.

Fisher, F. M., 1983, *Disequilibrium Foundations of Equilibrium Economics*, Cambridge: Cambridge University Press.

Fratini, S. M., 2009, 'Real Wicksell Effect, Demand for Capital and Stability', unpublished paper presented at Centro Sraffa, Rome.

Garegnani, P., 1970, 'Heterogeneous capital, the production function and the theory of distribution', *Review of Economic Studies,* 407–36.

Garegnani, P., 1978–79, 'Notes on consumption, investment and effective demand. Part I', *Cambridge Journal of Economics*, 2(4), 335–53, and 'Notes on consumption, investment and effective demand. Part II', *Cambridge Journal of Economics*, 3: 1, 63–82: as reprinted in John Eatwell, Murray Milgate, eds, *Keynes's Economics and the Theory of Value and Distribution*, New York: Oxford University Press, 1983, pp. 21–69.

Garegnani, P., 1990, 'Quantity of capital', in J. Eatwell, M. Milgate, P. Newman, eds, *The New Palgrave: Capital Theory*, London: Macmillan, pp. 1–78.

Han, Z., and B. Schefold, 2006, 'An empirical investigation of paradoxes: Reswitching and reverse capital deepening in capital theory', *Cambridge Journal of Economics* 30: 5, 737–65.

Hicks, J., 1965, *Capital and Growth*, Oxford: Clarendon Press.

Hicks, J., 1973, *Capital and Time*, Oxford: Clarendon Press.

Jorgenson, D.W., 1963, 'Capital theory and investment behavior', *American Economic Review*, 53(2), 247–59.

Kurz, H. D., ed., 2000, *Critical Essays on Piero Sraffa's Legacy in Economics*, Cambridge, Cambridge University Press.

Laing, N. F., 1991, 'The likelihood of capital-reversing and double-switching', *Bulletin of Economic Research*, 43, 179–88.

Mainwaring, L., and I. Steedman, 2000, 'On the probability of re-switching and capital reversing in a two-sector Sraffian model', in Kurz (2000), pp. 323–54.

Malinvaud, E., 1986, 'Pure profits as forced saving', *Scandinavian Journal of Economics* 88: 1, 109–30.

Ochoa, E., 1989, 'Values, prices and wage-profit curves in the US economy', *Cambridge Journal of Economics*, 13, 413–29.

Petri, F., 2004, *General Equilibrium, Capital and Macroeconomics. A Key to Recent Controversies in Equilibrium Theory*, Cheltenham: Edward Elgar.

Petrović, P., 1991, 'Shape of a Wage-Profit Curve, Some Methodology and Empirical Evidence', *Metroeconomica*, 42: 2 (June), 93–112.

Potestio, P., 2010, '"Perverse cases" and the debate on neo-classical theory of distribution: recent contributions on an open issue', in John Vint, J. Stanley Metcalfe, Heinz D. Kurz, Neri Salvadori and Paul A. Samuelson, eds, *Economic Theory and Economic Thought*, London: Routledge, pp. 138–60.

Salvadori, N., 2000, 'Comment on Mainwaring and Steedman', in Kurz (2000), pp. 354–8.

Salvadori, N., and I. Steedman, 1988, 'No Reswitching? No Switching!', *Cambridge Journal of Economics*, 12, 483–6.

Samuelson, P. A., 1962, 'Parable and Realism in Capital Theory: The Surrogate Production Function', *Review of Economic Studies*, 29, 193–206.

Schefold, B., 1976, 'Relative prices as a function of the rate of profit: a mathematical note', *Zeitschrift für Nationalökonomie* (*Journal of Economics*), 36, 21–48; repr. in Schefold 1997a, pp. 46–75.

Schefold, B., 1997a, *Normal Prices, Technical Change and Accumulation*, London: Macmillan.

Schefold, B., 1997b, 'Classical Theory and Intertemporal Equilibrium', in Schefold 1997a, pp. 425–501.

Schefold, B., 2010, 'Families of strongly curved and of nearly linear wage curves: a contribution to the debate about the surrogate production function', in John Vint, J. S. Metcalfe, Heinz D. Kurz, N. Salvadori and Paul A. Samuelson, eds, *Economic Theory and Economic Thought*, London: Routledge, pp. 117–37.

Sraffa, P., 1960, *Production of Commodities by Means of Commodities*, Cambridge: Cambridge University Press.

Tsoufildis, L., and T. Maniatis, 2002, 'Values, prices of production and market prices: some more evidence from the Greek economy', *Cambridge Journal of Economics*, 26, 359–69.