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Forward: Part I

Preface

It is fair to say that Joint Supply or Supply at joint cost belongs to the Neo-classical terminology and reflects its non-linear treatment of production functions, while Joint Production is typically associated with the Classical approach and its modern version, the linear production models. In this presentation, however, for simplicity, the two terms will often be interchanged. Until recently, the topic has been confined to article sections and book chapters, often to appendices, dedicated to the applications and extensions of models without it. This short tract by Marco Fanno, available in English for the first time over 80 years after its original publication as a supplement to the *Giornale degli Economisti*, is one of the few exceptions, and the only for his times, of a whole monograph devoted to it. From the title, it is a treatise in the Neo-classical tradition. Joint Production, on the other hand, has aroused a renewed interest in the 1970s and 1980s in a quite different context, in connection with the examination of von Neumann, Sraffa and neo-Austrian models of prices, capital and growth, with a "spillover" on the treatment of Marxian theory of labour values and prices. Fanno's treatise reminds us that the history of the subject is much longer, and is well documented as a sequel of sometime hot disputes between adherents of classical and Neo-classical schools (but also among members of the same school).

As the recent debate pointed out, the mere presence of joint production (even to a limited extent) changes some of the basic properties of prices and equilibrium as described in the Classical models. The issue becomes, then, to assess its relevance. Is this "empirical relevance", i.e. does this lie in the fact that joint production is the general empirical property of observed production processes? Or is its relevance in the fact that Joint Production provides an analytical method to handle some generalised pricing problem? Answers to these questions reflect two different conceptions, for on one side we would have a "description", on the other a "method". The issue has presented itself in about the same terms also at the turn of the century. Fanno gives us a refreshing perspective over it.

The history of the periodically reviving interest for joint production has something in particular to offer to our reflection. I surmise that various debates never reached a generally agreed solution because too many, and not only purely analytical, questions are at stake. However, though themes are cyclically the same; perspectives are changing, so that Joint Production offers a good case study for comparing modelling strategies and for appraising theoretical motivations. Perhaps, one does not want to go as far as to say that the difficulties encountered in its treatment played a

significant role in the abandonment of the Classical approach to the theory of value and distribution. Nevertheless, its treatment reveals aspects of method and heuristics, that "simpler" production models leave in the twightlight. [1] This is the main motivation for undertaking the present edition of this work of Fanno.

The perspective in *Contribution* is one of economic policy and regulation: The theoretical investigation is motivated by the purpose of arriving at prescriptions of industrial and fiscal policies, as they would be called with the modern names: Taxation, monopoly and its regulation, pricing in different market structures, are some of the topics discussed at length, sometimes laboriously. It is this underlying motivation that justifies, at least partially, the relatively little attention paid to the analysis of "existence of equilibrium solutions", an attention very limited even for the standards of mathematical economics at Fanno's times. One perceives that, in the time since elapsed, economics and in particular the theory of prices has developed into a purely theoretical exercise, where reference to immediate empirical cases for motivation has been reduced or altogether abandoned. Such an approach, with all its burden of implicit assessment of relevance and content, though born in the modern general equilibrium economics (Arrow-Debreu *style*), has spread to the most distant lines of thought.

In this essay Sect. 1 will sum up Fanno's motivations and interests in writing this *Contribution*, and will relate them to his research project as well as the background of his times. Sect. 2 will discuss the definition of *supply at joint cost*, he selected, in relation to other definitions used in contemporary literature. In particular, the debate on railway pricing will be reviewed, where a distillation of the ambiguities in the literature on joint production emerged so that it supplies a background to understand some of the issues treated in Fanno's work. Sect. 3 elaborates on the analytical method in Contribution, and suggests a relationship with some features of Hicks' approach in *Value and Capital*. In the final section, a discussion of the issues of determinateness and existence of equilibrium prices brings us back to the themes that surfaced and dominated the attention in the modern debate.

Sect. 1: On Fanno and his time.

At the time Fanno wrote this essay, the school of Italian economic thought, once on the frontier with such thinkers as Salimbeni, Galiani and the Illuminists, had fully come back to a high figure. To quote Schumpeter: "The most benevolent observer could not have paid any compliments to Italian economists in the early 1870s; the most malevolent observers could not have denied that it was second to none by 1914".[2] This happens to be the year of . of *Contribution to the Theory of Joint Supply*.

[1] see H. Kurz (1986).
[2] Schumpter (1954), p. 855.

Schumpeter kept a close eye on his contemporary Italian economists. He was well read in the Italian economics literature, like all economists of the Austrian School, and the *History of Economic Analysis* gives precious hints. In his reconstruction, the landscape of Italian thought had three successive layers, corresponding to three time periods and leading figures. The period of the forerunners (e.g. Ferrara, Graziani) is followed by the period dominated by the figure of M. Pantaleoni, and finally by the time of V. Pareto. Although working under the close protection of Loria, at the beginning of his career, Fanno feels the influence exerted on the Italian economists by M. Pantaleoni, directing their reflections towards topics in applied economics and fiscal policy. He is in a group with such people as Antonelli˙ and E. Barone, whose contribution on the use of general equilibrium analysis for central planning gained him a lasting international recognition. Only later, would Fanno get closer to the Paretian school, when this became the dominant school of thought in Italy, but he always remained a loose partner.

Contribution belongs to an intermediate stage between these two phases in Fanno's personal evolution and in the evolution of the Italian economists milieu. The two processes largely mirror each other, notwithstanding some peculiarities in Fanno's peculiar intellectual *iter*. However, "even independently of Pareto, Italian economics attained a high level in a variety of lines and in all applied fields".[3]

Variety of interests, together with a certain tendency to heterodoxy[4], is the distinctive character of Fanno's scientific production. His works can be grouped under three principal headings, which correspond different phases in his research. He begins with the theory of development, and in particular the theory of colonisation. His interests move on to the theory of money and credit, where he focuses on the Wicksellian theme of the relation between discount rate and long run rate of profit (the natural rate) and on its influence on inflationary and deflationary processes. The conclusive moment of Fanno's meditations is represented by *Teoria delle Fluttuazioni Economiche*[5], that of his works which received most international attention. There, economic fluctuations are treated as persistent phenomena of endogenous origin, ignited by external shocks (an idea that has some remarkable proximity with R.

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[3]Schumpter, *ibid.*, p. 855. The single most important representative of this line of thought, conjugating theory with applied analysis, is obviously L. Einaudi. Actually, the Italian landscape was more rugged than Schumpeter says, as Bagiotti observes : "Fanno's Italian contemporaries, with various degrees of subtelty and individuality, stood between the tendencies of historical or pure economics (with a few lags and revivals), and classical Marxist criticism. Pure economics was processed in terms of both causalism and functionalism, with various functionalistic factions forming around the personality and the writings of Pareto". And specifically "Fanno, whose rapid career had been sponsored by Loria, reflects some of his tastes at his beginnings. But other inspirations, other themes, and the personality of Pantaleoni, versatile and extemporaneous, constrained him to turn to profounder theoretical considerations." (All citations from Bagiotti (1966), pp.XXI-XXII.) For more details on the history of Italian thought see Finoia (1980).

[4] As pointed out by Legris, (Legris (1994).

[5] Bagiotti (1966). For Fanno's model of fluctuation, see the contribution by Montesano who associates it with the exogenistic model of Frisch, while Cozzi gives an endogenistic interpretation. both of them in Bagiotti, (1966). Halevi (1987) accepts Montesano's interpretation..

Frisch's theory). It is, however, remarkable that Fanno's theory is based upon the interaction of the monetary and real mechanisms, while the theories of fluctuations available at the time (and now, one can say) were unilaterally referring to either mechanism [6].

There is also a fourth group of contributions, written in a vein closer to pure theoretical speculation, which represent periodic but temporarily sparse incursions into the theoretical realm rather than a systematically pursued research interest. The present *Contribution* is one of them , and is accompanied by a co-ordinated title, *Contributo alla teoria economica dei beni succedanei*, (1926), and two more equally short monographs.[7]. Surely not the largest share in Fanno's production is devoted to pure economics; empirically motivated research and applied work has always exerted a great attraction on him.

Schumpeter, who was educated in the German-speaking milieu of Economics, with its heated debates between the Historical school and the Austrian ("Logical") school, could not fail to appreciate the distinctive feature of the Italian economists of the turn of the century: their ability to marry economic theory and economic history, analysis and application. One in that school, Fanno is an empirical or applied economist of high rank, and this shows even when he does theory as in *Contribution*.

This gives a key to understand the motivations for and the approach taken in this work.

Sect. 2: *Fanno and Marshall.*

As a specialised monograph, *Contribution to the Theory of Joint Supply* is an exception in the literature on production and supply. Its existence reminds us that the intricacies of the topic did received attention in both Classical and Neo-classical literature. A casual perusal of the literature, at least in English, shows that by the time Fanno set down to write this work. Joint Supply, though still presented as a "complication" of the standard cases of production, had filtered out of Treatises and found its way into textbooks. Two notable examples are the works of Nicholson and Flux, which did not entertain the ambitious, all-encompassing plan of Alfred Marshall's *Principles.*

Marshall's *Principles* provides the point of departure and the theoretical backdrop of Fanno's own investigation, which is therefore fully inserted in the Neo-classical approach. One can re-construct *two motivations* for the undertaking. The first

[6] See Bagiotti (1966), for a description of the works of Fanno. This book provides a number of contributions assessing Fanno's work. One chapter, " Public Goods and Joint Production", by C.S. Shoup, is devoted to Fanno's theory of Joint Cost. See also Bartoli (1981); Manfredini Gasparetto (1992).

[7] These are: "Correlazioni tra prezzi e curve statistiche di domanda e offerta" (1932), and "Della identita' di alcuni teoremi di economia pura" (1934).

is to fill in conceptual gaps in the existing analyses (the reference is explicitly also to Marshall) and to provide a general treatment of joint supply; next, to integrate it into a general treatment of the "pricing problem"[8]. The latter is the ultimate theoretical construction which is aptly called "organic" (Fanno, 1924). Its realisation is planned in successive stages, by implementing a procedure of progressively adding successive "theoretical bricks", which recalls the analytical structure in Walras' *Elements*, and the techniques of classification of the biological sciences at the origins.

To Fanno, *some* of the goods are complementary in production, and joint supply is *not* a general case that the literature should have treated at greater length but neglected. Goods produced in this regime span one group in a two-parameter family classified according to the principles of complementarity and rivalry, on one side, and demand and supply, on the other[9]. I.e., goods can be jointly supplied or jointly demanded, *and* can be complementary or substitutes to each other. Only the first ones are the subject of this monograph, which therefore offers a partial treatment of the general pricing problem. Nevertheless, the analysis of the general movements of prices and quantities through the interconnections between their markets is Fanno's end objective. Before attacking this, one needs to consider all the other, "dual goods" in the matrix classification above: There will be a separate study of the correlations existing between the price behaviours of goods in each group, under the assumption of having de-coupled the sets of price equations that belong to distinct groups of goods[10].

Goods' classification is a rhetorical device to simplify the treatment, as their qualitative spectrum varies gradually; borderlines are fuzzy. In the second phase, this "logical" device has to be abandoned to "invert the initial process so as to reconstruct the system as a single whole". (Fanno, *ibid.*, p. *13*). Concatenation between the two analytical phases is simplistic as the latter builds upon the *imperfect* and *incomplete* laws derived in the former. The realisation of the overall objective requires, in the final phase, the co-ordination of the general theory of determination of relative prices, with the theory of monetary circulation. This theme (rather a preoccupation), constantly at the centre of the theoretical reflection of Fanno on economic dynamics, finds its way here too. The interplay between monetary and real variables, elaborated from Wicksell, links *Contribution* to the theory of business fluctuations, which will soon come to dominate his future theoretical reflection. There, it plays the role of driving engine of the system dynamics.

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[8]Fanno in his Obituary for Marshall, (1925), p. 173. On the influence of Marshall, see the illuminating words in A. Legris (1994).

[9] "A classification which no does not reflect real objective differences, but like all classifications it follows the kind of clear and precise qualitative dividing lines that simply do not exist as such in the real world", Fanno, *ibid.*, p. *12*.

[10] "I shall be studying each of these four groups separately, looking at how price equilibrium is established within each group and at how each individual price acts and reacts on others on the group", *ibid.*, p.

Fanno realised only the first of these planned phases of research, and even this only partly. The counterpart to *Contribution*, dedicated to substitutable goods, found eventually the light in 1926; the overall plan was never accomplished. His interests, which we see already clearly directed toward dynamics, shifted to the analysis of business cycles, which must have appeared more promising to an empirically-minded economist.

The starting point of *Contribution* is the remark that existing treatments of joint supply consider only special cases: one isolated market, which is in competitive regime and closed to international trade. Fanno sets out to extend the received theory to cover monopolistic regimes and internationally open markets. This variety of combinations of market structures and inter market relations is first analysed by means of the *partial equilibrium* approach, focusing on the (closed/open, competitive/monopolistic) market for an isolated good. By its very nature, however, the working of the price mechanism of jointly produced goods cannot be fully understood in isolation from one another, and therefore it is planned to be further generalised to any number of interrelated markets.

Thus, there is a twofold reason for the analysis in *Contribution* to be part of a planned complex theoretical architecture: it refers to one type of price correlation; *and* it resorts to Marshall's technique to derive building blocks. For Fanno just like for Marshall, the treatment of joint supply gives one tool to construct a more general model, for in the small it captures the intricacies of a construction that has to reflect the working of all markets. For both, the problem arises of the relationship with the general equilibrium approach of Walras and Pareto. In a sense, the problem is even more central to an understanding of Fanno's mediation between the English and the Italian Schools.

One need not go all the way with Schumpeter, who tends to over-stress the Marshallian inspiration in Fanno's work. This suggestion that their treatment of joint supply be conceptually complementarity hides important differences, perhaps not perceived clearly by Fanno himself [11].

Marshall devotes most of his attention to the case of attributable costs where, due to substitutability or rivarly in production, cost can be surely imputed to individual goods. For him, to put it in Schumpeter's words, Joint Supply might be the *ladder* to General Equilibrium analysis[12], as built upon the principle of substitution. It "opens up one of the several possible roads toward the recognition of the universal interdependence of economic quantities." [13]

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[11] I agree with Bagiotti's remark that Schumpeter has over stressed the link between joint supply and general equilibrium (Bagiotti, 1966, p. XXII).

[12] Refer to quote of Marshall's Principles, Appendix to Book V.

[13] "From the standpoint and within the precincts of partial analysis, this universal interdependence was demonstrated by numerous investigations into the theory of joint and composite demand and supply and of the values of related commodities which produced some of the most illuminating passages of Book V of the *Principles* and which further developed by Edgeworth. In fact, it stands to

Fanno, on the other hand, treats joint supply as one manifestation of the general phenomenon of functional interdependence˙, and begins his investigation into the dynamic working of the price mechanism with this treated as a "case". He shows how firm is his belief in the correlation between all prices and in universal interdependence of all markets when he anticipates that the laws regulating equilibrium displacement will result from the networking of markets. His ambitious plan is of conjugating Marshall's with Walras' (and Pareto's) viewpoints, in an operation which might have appeared theoretically suspicious at the time[14], but had a fundamentally operational motivation. If each component had a role in his synthesis, the latter provided the "vision".

Marshall treats joint supply to show how his approach to the imputability of production costs could be extended to cover that case. For Fanno, borrowing the Partial Equilibrium approach is a "practical" means to *de-code* general interdependence. It is in this specific sense that "The extent to which partial and general equilibrium analysis may cooperate shows up well in Fanno's 1914 and 1926 contributions"[15], captures Fanno's attitude in *Contribution*. In the seemingly simplified mental experiments constructed in *vitro* in this little treatise, he saw (or, rather, perceived) the possibility of finding a path to reveal the building blocks of more general laws. This belief in a sort of linear and additive relation between partial/small and general/big, upon which a practical approach to general equilibrium could be founded, proved historically to be ill founded, as the developments of the General Equilibrium literature had soon to show.

Sect. 3: *Joint supply.*

A glance at the 1913 and 1914 issues of the *Quarterly Journal of Economics*, suggests that there might be a second, not less important, motivation for the theoretical quest in Fanno's *Contribution*. I am referring here to the debate on railway fares and freight tariffs which saw Pigou and Taussig as opponents then, but was really the culmination of an abundant specialised literature that blossomed at the end of the century. The *QJE* dispute finds an echo in the opening section of Fanno's work, in his discussion of the choice of an appropriate definition for *supply at joint cost*.

On the *QJE pages*, the issue of joint supply was tackled in relation with a practical problem: devising an appropriate pricing scheme for *services jointly offered* (like the transport of different minerals to different final purchasers) and the instance of

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reason that the comprehensive but gaunt and *colorless* idea of the universal interdependence (..) - and so easily provokes sneers about everythind depending upon everything else- can be brought home and made alive to many by means of concrete cases about the relations between the values of beef and mutton or again of tea and sugar. (..) And this can be done by violating the restrictions inherent in the method of partial analysis." (Schumpeter, *ibid.*, pp. 995-6).

[14] "In Italy, where the adherents to the theory of economic equilibrium began to form faction, the gist of the theory of these monographs had to seem somewhat spurious." (Bagiotti, *ibid.*, p. XXII).

[15] Schumpeter, *ibid.*, p. 996, fnote 12.

railway transportation offered the immediate inspiration and the practical ground of observation. The assessment of the presence of monopolistic behaviours and of regulatory implications were the focus of the exchange between Taussig and Pigou. Beyond the apparently conciliatory words of Pigou bowing to the authority of Taussig, their views and attitudes were profoundly different, the former looking for a clear cut "technological" definition (one would say an axiomatic statement), the latter for an economic intuition.

To Taussig, the conditions for joint production of goods and services arise whenever there is large fixed capital in operation, i.e. large plants and equipment stocks, which are able to generate a plurality of output over the same time period or at the same date. Such form of indivisibility at the level of capital stock translates in the impossibility of reckoning the individual cost components for the various items in the output bundle. In fact, in the operation of Railways, "The plant -almost all of it- is used for the traffic as a whole, without the possibility of allocating any part of this constituent part of supply price to one or another item or class of traffic. (...) Now I am convinced that the conditions of supply for railway transport are preponderantly joint."[16] Evaluating the contribution of capital investment to the individual components in a vector of output flows has a dual problem, that of evaluating the contribution of an investment over its (economic and/or physical) lifetime to the individual components of a time series of output.

If “horizontal” joint production is typically associated with the presence of large equipment and production facilities, then it derives from the economic decision to undertake the corresponding investment. The regime of joint cost production, in other words, is a result of optimising decisions taken after assessing the relative convenience or profitability of producing with different types (sizes) of equipment. In other words, according to Taussig, it is not the result of a technological indivisibility.

It is Pigou’s view that, on the contrary, one should talk of joint cost only "whereas it is impossible to increase the supply of cotton fibre without at the same time increasing the supply (...) of cotton seed, it is not possible to increase the supply of transport services for copper without at the same time increasing the supply of transport services for coal."[17]

The implications of accepting Taussig’s view, both for positive as well as for normative economics, are far reaching, and are clearly stated by Pigou. If the services of transporting passengers and goods were produced jointly, “even under conditions of free competition, different rates would normally be charged for them.” On the other hand, charging different tariffs for what was the same service, would represent discriminatory behaviour, that could only be based upon monopolistic power. In fact, " If classification , "discriminating rates", charges based on "value of service", are the results essentially of monopoly, as professor Pigou contends, they must go with the

[16] *QJE*, 1913, p. 537.
[17] *QJE*, 1913, p.691.

inhibition of monopoly exploitation. But if they are the results mainly or largely of joint supply, they may remain even tho the railways are compelled to conduct their traffic on the same principles as competitive industries"(as Taussig put it [18]). The implications for what we would now call industrial and in particular regulatory policies (e.g. anti-monopoly legislation) are the fundamental motivation behind a debate which at first sight seems to revolve around the purely theoretical issue of an appropriate definition. The exchange, however, reveals two different conceptions and an ambiguity which runs through the history of the subject, all the way down to recent years. This is due to the simultaneous presence of a technological and an economic definition of joint supply.

Pigou tried to clear it up by introducing a distinction between *supply at joint cost* and *supply at common* (or *shared*) *cost*. The former reflects the technical conditions of production (i.e. properties of the production functions) and broadly corresponds to the modern usage when we talk of intrinsic joint production. The latter reflects the economic convenience that leads to organising production in such a way as to obtain a vector of outputs, i.e. a form of product diversification. But Pigou's attempt at clarifying the terms of the question is only partly successful. Though rigorous, his distinction is often blurred when need comes to apply it to practical cases.

Joint cost in the sense of Pigou is certainly common cost, but not vice versa[19]. Having made this remark, Fanno re-phrases the distinction as one between a strict (Pigou's) and a broad (Taussig's) definition. Taking side with Pigou, he insists on reserving joint cost for the cases where it "occurs spontaneously", while in the broader definition, it emerges "purely (as) a result of considerations of convenience"[20].

Taussig, however, was not taking advantage of a definitional ambiguity when speaking of a *principle of joint cost* which "may be applicable even tho a supply of one does not necessarily entail the supply of the other" good.[21].He is touching upon a real, theoretical and operational difficulty, which only an altogether different, axiomatic, approach could overcome. Pigou and Fanno took "wheat and straw" as their paradigmatic example. As a shortcoming of their logical rigour, joint supply was restricted to what we call these days "intrinsic joint supply", perhaps the less economically interesting case. Pigou and Fanno with him left out fixed capital, which is the case that, with a certain amount of confusion, Taussig had in mind. He was stressing a "horizontal dimension" of joint production associated with fixed capital, which is dual to the "vertical dimension" at the centre of Austrian capital theory. Pointing out that both of them stem from the fact that there are economic decisions

[18] *QJE*, 1913, p. 538.
[19] In Taussig's definition "supply at joint cost includes supply of goods or services that can take place only connectedly and supply of goods or services which could be produced separately". Fanno, *ibidem*, p.*22*.
[20] .Fanno, *ibidem*, pp. *21-22*.
[21] Taussig, *QJE*, 1914, p.684

behind production with fixed capital goods, Taussig was confusedly thinking of what came to be known as the problem of Akerman (and Wicksell).

Fixed capital goods are producers' goods whose useful lifetime is longer than the unit production cycle conventionally chosen. Calculating the cost and the return from their operation poses an imputation problem. At the end of each cycle (e.g. the crop rotation year), they re-appear as only “partly worn out machines", until they are finally scrapped. Therefore, at any date in a given utilisation profile, the "machine" enters as input at the beginning and must be reckoned as output (of one-period older machine) at the end, together with the intermediate or final commodity (or commodities) it was employed to produce. This is one aspect of the evaluation problem, its “accounting side”, as the operating lifetimes of machines are given. In this form it was tackled by the Classics.

However, the economic lifetime of a machine is the result of a choice: depending on the overall economic conditions, it may turn out to be convenient disposing of a machine before it has become physically useless. Hence, with lifetime an endogenous variable, only producers' goods which are chosen to be operated longer than one unit period, qualify as fixed capital goods, and such unit period of reckoning becomes an endogenous variable, too. It becomes the shortest time period to produce identifiable output with capital goods. In accounting, on the contrary, the set of fixed capital goods (and the accounting period) are taken as a datum, being the result of past decisions. Thus, if we abandon the accounting viewpoint, the analysis of fixed capital is always associated with that of economic decisions made in respect of technological constraints.

If we can re-read it with the modern eyes, after von Neumann’s model of growth and prices, Taussig’s contention was not ill-founded, after all. The operation of fixed capital entails *per se* aspects of joint production, that could (should) be treated on the same footing as intrinsic joint production. Could we re-phrase Taussig's argument this way, he surely would be on the better side with respect to Pigou and Fanno. As worked out by von Neumann and Sraffa, well after Fanno wrote this monograph, joint production has become an analytical method[22]. The method of joint production is the descendent of supply at shared cost, just like intrinsic joint production claims Pigou’s joint cost as ancestor. However, they do not relate to each other as general to particular cases do.

They refer to two distinct classes of phenomena, and the attention of the economists has been dwindling from one to the other. Edgeworth's definition in *The Palgrave* is an authoritative example of a wording that would have agreed with both, Pigou and Taussig, when he says that joint supply takes place " when two or more things are produced by one and the same process, so that the expenses of producing them altogether are not greater than the expenses of producing one of them alone".

[22]The problem originates much earlier, and found a treatment in J. Rae (1834) and later in Akermann's much celebrated doctoral dissertation, reviewed by Wicksell in an appendix of his *Lectures* (1934).

The Pigou-Taussig debate (and I dare say, of Fanno's immediately subsequent *Contribution*[23]) did not help to resolve the opposition between *method* and statement of a *technological property*. Moreover, if the former is relevant as far as production with capital is considered; the latter may or may not be empirically relevant[24], or it may be theoretically relevant and practically irrelevant[25]. Arguments supporting the relevance of joint production for the two conceptions are different.

I believe that neither Classics nor Neoclassics ever made up their minds on this. The reason for this lies in the fact that neither line of thought adopted a deductive approach, where technology is axiomatized as part of primitive data. They both start from intuitive observations, instead, and take realised technology. In this ex post view, the distinction between the two classes of joint production (which must be based upon their technological or economic "causes") becomes blurred. The mere fact that a certain technological operation is observed finds its “necessity” in its existence. Technological necessity and economic convenience cannot be separated from one another.

This is why Taussig never yielded to Pigou's otherwise compelling arguments. Axiomatic was any way beyond theoretical mentality of early 1900 economics. It was next step, to be made by J. von Neumann. Not everything was resolved, however, as we still meet with Sraffa's sentence: "Joint production is the genus of which fixed capital is the leading species"[26] where again “method” and “empirical description” are assimilated. At the other end of the century, we have not made much progress towards clarification.

Sect. 4: *Fanno and Hicks.*

Fanno’s adoption of the restricted definition advocated by Pigou is a shortcut to identify and isolate one group of phenomena, joint production of the "intrinsic" or "technological" type. It does not imply the decision of axiomatizing (Fanno could not be more remote from this idea), nor does it carry Jevon’s implicit value judgement, that this sort of joint production is the general phenomenon in production.·The study of the conditions for the establishment of equilibrium prices for joint products belongs to a different plan: providing benchmarks for assessing observable economic behaviours and evaluating policy measures. Therefore, via comparative statics, the core of the monograph states a set of propositions on the effects of disturbances on the supply (due to taxes or subsidies) and/or on demand sides (determining demand shifts). These

[23]As acknowledged by Edgeworth in his comments to Fanno's work, in Edgeworth (1915).
[24]As suggested by Jevons, Edgeworth himself, and more recently by the neo Ricardians e.g. Steedman (1984), Kurz (1986), Schefold (1987), to name some of them. Moreover, according to Kurz, results holding for the case of non joint production are not relevant for they cannot be carried over to this "more general case".
[25]This position could be associated with the name of Marshall, Book V, who suggests that "it is hard to find examples of J.P. where the quantities of individual commodities could not be changed at the margin without affecting the quantity of the jointly produced good”.
[26]Sraffa (1960), p.63

are not simple “corollaries”: “To know the laws of prices really well, it does not suffice to affirm that all prices are connected among themselves. It does not suffice to furnish the system of a group of equations in which the prices of all goods and services function as simultaneous determinants. In addition, it behoves to consider the nature of the relationships that exist between prices - the direction and the intensity into which various prices move one with respect to the other - the modes in which the various groups of prices are connected. Concerning all this the school of Lausanne has until now kept silence.”[27]

The grandeur of the vision proposed by Walras and Pareto, with “a vast, complex, tightly-knit web of relations holding all prices together by their common chain of solidarity", makes also its *sterility*. To Fanno, the theory of General Equilibrium still “ furnished only a scheme rather than a full co-ordinate system for the laws of prices.” A grand plan had been laid down for people's consideration, but its ultimate purpose remained elusive. Fanno's concern for a practical re-construction of price theory shows how advanced was the Italian thought of the time, as it already perceived the need to go beyond general equilibrium theory as it stood at the time.

At Fanno's times, three possibilities were open, in principle, for further investigation: 1. the first was to deepen the mathematical understanding of the statements and theorems, in that theory, accounting for the working of the “invisible hand”; 2. the second entailed further investigating the computability of equilibrium solutions, which could provide a benchmark in descriptive and/or normative economics; 3. the last one was to work out the dynamical laws that could be used for "economic prediction", forecasting in a broad sense. To E. Barone, among the Italians, the task of progressing beyond the initial statements of the grand vision meant following the second of the alternatives above. He demonstrated how Walrasian (and Paretian) analysis could be used for planning by showing the equivalence of the latter with computing the solutions to a system of general equilibrium equations. The elaboration of the theorems on the existence of equilibrium, as the manifestation of the working of the invisible hand through the pricing problem, had to wait some 20 years to be tackled in earnest, in Vienna first, and finally developed in the US.

Fanno tried out the last of the above tracks, and along with him went a number of Italian mathematical economists who were active until the 40s, Giulio La Volpe among them. In his investigation, Fanno deploys a method that, later, was going to be called the static equilibrium method by Hicks. He makes the twofold assumption, that there is a known equilibrium to start from (without investigating the mathematical conditions of its existence), and that such equilibrium is described by the equations of his model: The sentence : "Consider a given equilibrium and suppose that the initial conditions of it change consecutively one by one"[28] reminds of Walras' assumption of his model describing "an equilibrium ab ovo". Fanno, then, proceeds to investigate the working of the Invisible Hand not in establishing consistency among individual choices,

[27] Fanno, *ibidem,* p.*7*
[28] Fanno, *ibidem,* p.*41*.

but in re-creating harmony between them. A static method was used to tackle dynamic issues.

Asserting the existence of an equilibrium without proof, is obviously insufficient and it no longer appears legitimate to our eyes. The assumption, however, is purely instrumental to focus on the adjustment processes and on the laws of equilibrium displacement, and it may generate a knowledge basis for prediction and normative prescription. This is the result that Fanno was striving to obtain; that assumption seemed to be the price for pay for it. Now, the shortcoming of such procedure is that the resulting analysis has to remain "local" and may consider only shifts in parameters and environment conditions which do not lead "too far away" from the original equilibrium. This, however, is what Fanno's analysis intends to do.

Even before the Arrow-Debreu-McKenzie codification, General Equilibrium theorists were aware of these limitations, and accepted them for a purpose, rather than simply for its mathematical inability of handling the general problem[29]. Global analysis of system properties was traded off for a more desirable result, obtaining dynamic laws that in principle could be explicitly derived, though perhaps only qualitatively·. At the time, to build a general analysis only meant relying upon the principle of universal functional interdependence. Fanno tried to marry Marshall with Walras and Pareto; some time later, Hicks attempted the compromise with Keynes's thought which prepared for the Neo-classical Synthesis. Fanno's inspiration is similar to the one behind Hicks' project in *Value and Capital*.

Hicks' defence against the allegation that his models they do not care about supplying existence proofs, was that they were mental laboratories constructed to carry on conceptual experiments. Reading Fanno's *Contribution* conveys very much the same impression. Fanno, too, is building an artificial laboratory for experiments that our discipline could not carry out at the time otherwise. New technologies available at reasonable cost and on a mass basis seem now to promise different, life scale laboratories for our experiments of the future.

The reader may or may not believe the theoretical and/or empirical relevance of Joint Production. However, the debate about Joint Production at the beginning of this century offers an unusual perspective on the unfolding of a research project, that originated in one tentative of exiting from the theoretical confines of GEE. In a sharp depart from this, much of the recent debate has revolved around those issues of equilibrium existence and determinateness which are typical concern of the modern general equilibrium theory.

[29]Obviously, this is an ex-post rationalization, for none of the economists involved had the mathematical apparatus nor skill to handle the problem as it should have been handled, and actually the appropriate mathemtaical tools, which eventually had to find employment to the purpose (with von Neumann, A. Wald and others), had just been created. Brouwer's fixed point theorem was published in 1912!

Sect. 5: *Equilibrium Existence and determinacy.*

True, though Fanno is careful in dealing with two interdependent markets for jointly produced commodities, he boldly bypasses the mathematical issues arising in the general case with *n* markets. He seems to believe that conditions for existence for two commodities carried over to n commodities without relevant modifications. The recent debate has shown that this is not true and has focused on the role of "demand" when the system of price equations is not fully determinate. One tends to re-read old treatments of joint production with modern eyes, generating misunderstandings that need to be spelled out. This can be done in the language of production processes analysis of linear economics.

In standard Neo-classical approach, the number of production processes available ex-ante is always infinite due to the possibility of continuous substitution between inputs along smooth production functions *and* variable returns to scale. With each producible level of output is associated a process (possibly a set of processes) with a unit cost or supply price. As they span a whole set, technical supply conditions alone are not able to determine prices for any commodity. They yield "too many" price equations.

On the other hand, the number of producible goods is finite and therefore always smaller than that of technological processes. Thus, as determination of equilibrium prices cannot be separated from the determination of output levels, "demand conditions" are introduced to pin down which process(es) will be activated (at the chosen level of output), *and* the corresponding vector(s) of equilibrium prices. “Solving” the market equilibrium equation between demand and supply, implies a notional search procedure through an infinite set of supply and an infinite set of demand values·, to find at least one demand and supply relation that hold as equality. In equilibrium, equations holding true are (almost) never fewer than the prices to be determined.

Therefore, contrary to a frequently entertained opinion, demand is not brought into the Neo-classical theory of production in order to supply some “missing equations”. Its role is to close a system of relations which, anyway, remains in general mathematically over determined.[30] Such over-determinacy implies that the set of activated processes, the equilibrium technique, is not identified uniquely: several techniques are normally eligible for the production of the same bunch of goods. Qualifications to this general rule must be introduced if we assume the special case of linear fixed coefficient production functions. The principal one is that the choice of the set of processes to produce any one good in equilibrium becomes independent from both the scale of production and the composition of final demand. However, it remains true that unit costs are uniquely determined by cost minimising considerations, and that equilibrium prices are independent of final demand composition. This is nothing but the

[30] Actually, the introduction of demand would make the situation worse!

once celebrated Non-Substitution Theorem, a particular case of General Equilibrium pricing model.

In Neo-classical analysis, therefore, demand plays a crucial role in determining the equilibrium values and the supporting structure of processes that can be activated (the realised equilibrium) due to the general non linearity of production functions. No opposition in principle between production with and without joint production was felt, as pointed out by Edgeworth: " The value of joint products is regarded by Mill as a "peculiar case" apparently because it varies with the demand for the products; (...) But with the wider conception of cost of production varying with the quantity produced (Sidgwick, *Pol. Econ*.,bk.ii ch.ii. S.7; Marshall, *Principles*, *Passim*) this difference between single and joint products disappears."[31] The complementary role assigned to demand functions demonstrate, in a static framework, the impossibility of de-coupling two parallel mechanisms of dynamic adjustment. When the conditions for the Non-Substitution Theorem hold, such de-coupling is possible and this reduces the role of demand to that of determining output proportions and levels only, within the constraints set by the availability of stocks. The conditions are, however, very restrictive. It is natural to the General Equilibrium Approach that quantities and prices be determined simultaneously, as interdependent variables. Therefore, the approach is at its best with non-linear demand and supply functions. This amounts to saying that variable returns to scale are considered the general case which admits the constant returns required by the Non-Substitution theorem, as a particular realisation. Joint production, even on a limited scale, shows that the latter is a necessary, but not sufficient, condition to de-couple the price from the quantity equations.

However, in the Neo-classical approach, the issue of pricing for Joint Products appears in a peculiar way. Let us admit only two joint products: there is one single production function, hence one supply function, *vis-a-vis* two separate demand functions, one for each good. No equation is missing; the difficulty is in that one supply function is missing, therefore, it is in principle impossible to derive two independent equilibrium equations to determine the required commodity prices. Demand is not introduced to meet the difficulties encountered by the Classics in the treatment of joint production (as suggested by Kurz, (1986)): demand is already there, in the Neo-classical model, before and independently of joint supply. Moreover, its presence "may help" but it does not solve the problem: how to allocate the aggregate cost given by a single supply function over two or more goods with distinct markets. The Neo-classical approach saw this as a problem of imputation of value of production back to its individual components, in other words as a problem of "disaggregation". This explains the discussion of the derived demand function in Marshall (Vol. V). It is even more clear in Fanno's treatment which is based upon an ingenious assumption of fixed proportions between component outputs[32].

[31] *Palgrave's Dictionary*, p. 485.
[32] Fanno, *ibidem* , p.*39*.

The Classical approach and its modern reformulation (the neo-Ricardian analysis) rely on a constant coefficient technology and encounter the possibility of finding fewer equations than price unknowns. In fact, with a discrete set of processes, each process has its own unique cost equation, and therefore any process with multiple outputs represents degrees of freedom. To obtain a determined problem, they need be matched by some other equations. The criticism levied against the Neo-classical approach can, therefore, be laid at the Classical doorsteps: the Classical system must be "closed" when *joint production* is introduced ; the Neo-classical system with production must be closed always, i.e. independently from the specific properties of the output vectors. From the mathematical viewpoint, the latter is tackling a more general problem than the former. The problem with the Classical approach to joint production can be seen in the following way.

Let the ex-ante technology matrix list more (or, at least as many) processes than goods to be produced. This is not sufficient to guarantee that there be at least one square submatrix for a "technique adopted in equilibrium": this may or may not happen, and on purely mathematical grounds, it can be argued that the matrix is likely to be rectangular.[33]. If it is square, the Classical price problem with joint production is solvable in the same way as the problem with single production, with prices uniquely determined by production conditions alone. Recalling the linearity of the technology (and some other standard assumptions), this shows that the Non Substitution Theorem may be generalised[34].

In this way, the issue of determining Classical production is transformed into the question: how likely is the equilibrium submatrix to be square? In this form it has been discussed in the recent neoRicardian literature[35].

The problem here, however, is located at the level of the data or technology matrix, not at the level of solutions. If the given technology matrix is rectangular (with m the number of rows/goods *greater* than n the number of columns/processes), the solution matrix cannot be square. Either more processes are added to the data set (the justification going with this procedure will have to provide the "missing elements" in the price theory); or demand has to be brought in; or, finally, some goods must be "dropped" from the description of production. That of "adding processes" is the solution attributed to the classics and appears to be adopted also by Sraffa. In a certain sense, adding demand conditions (though in a peculiar form) is von Neumann's solution. However, increasing the number of columns to construct a square data matrix, might be as good as decreasing the number of its rows. It can be argued that, if the number of goods exceeds the number of processes, it is "mathematically unlikely" that all goods receive a positive valuation in any equilibrium solution. To exclude the

[33] This is because there is no a priori relation between rows and columns. For a different opinion, see Bidard (1986).
[34] Goodwin and Punzo (1987), Punzo and Velupillai (1984).
[35] On the literature, see Salvadori and Steedman (1988); Schefold (1980), (1987).

possibility of free goods would require introducing peculiar assumptions on the structure of demand[36].

A group of "candidate free goods" is therefore any subset of goods containing (m - n) goods selected out of m producible goods. As there is a finite number of such groups, the treatment of the equilibrium pricing problem may proceed by searching through the set of square submatrices obtained by deleting the corresponding rows in the data matrix. Except for degeneracy, (at least) one of these matrices will be associated with equilibrium. This is the solution implemented in the treatment of ageing fixed capital goods. Assume that this is the only type of joint production and, moreover, that once installed machines (or better, “plants”) are not transferable. Then, for each “machine” and a fixed lifetime a square submatrix is obtained of processes that employ it, the last year in its life being signalled by its being treated as a candidate free good (and getting a zero coefficient). All submatrices are obtained by dropping out selected old machines from the list of commodities and are square; so square will be the equilibrium matrix, too.

In a certain sense, this alternative “solution” seems to be close to the Neo-classical treatment of Walras, Pareto and Hicks, that also Fanno follows in this *Contribution*. It is a rationalisation of the fact that these authors assume an equilibrium and study some of its properties. In equilibrium, the list of free goods is known.

This approach has both advantages and disadvantages. On one latter count, it hides the optimisation process in the choice of techniques, which instead emerges in an inequality approach where free goods are endogenously determined. On the former count, the linear treatment of joint production is traced back to the spectral analysis of linear operators, which is the fundamental algebraic theory of matrices·. Through the latter, pricing for joint production links up with the linear theory of dynamical systems.

[36] The assumption requires that relative quantities demanded be exactly equal to relative quantities produced.

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