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Clower on Axiomatics

Lionello F. Punzo
University of Siena

1. Themes

Robert Clower has a nice paper titled “Axiomatics in Economics”, published recently (1995), which prompts the title and defines the boundaries of my contribution. However his methodological interests are not at all recent. His epistemological position can be dated back to the candid statement that, in a monetary economy, goods are not exchanged against goods, they are exchanged against money. There is a group of papers produced all along many years, some collected in a book of recent release (Clower, 1996a), giving his personal reflections on problems of method and on issues of interpretation of current and past analytical developments.

These papers reveal an evolution in his thought so that I had to decide which position to take as representative, for I did not want to dwell on the exegetic of these changes. For my comments I will single out the most recent papers and will, moreover, consciously tend to emphasise differences between us for the sake of the argument. While I feel very close to his (most recent) position, I still see difficulties in stating it unambiguously due to its ramifications and ancient misunderstandings. Hence, I will act as the devil’s advocate in the spirit of improving communicability and teachability, as strongly advocated by Clower himself. Clower lifts up the lid of the classical Vase of Pandora, from which now lots of Hicks’ rabbits jump out¹, the existence of some of them to surprise Clower himself. I will list some of them in the sequel. Clower advocates inductivism in economics while not denying at the same time some use for Axiomatics. Which sort of axiomatics he rejects, and which sort he accepts, is the focus of most of my argument hereafter, while I will intentionally leave aside the issue of the desirability (and relative merits) of inductivism compared to the dominant deductivism. These may be the subject of some other contributions. In the views entertained by Clower, there is first a distinction between a (possibly “Applied” or “Empirical”) Theory and the “Pure” Theory, the former being the “fact-oriented creative mixture of intuition, causal empirical knowledge and seat-of-the-pants-logic that is found in virtually all applied economic analysis, and indeed, virtually everything called economics before 1950”. Pure theory in economics, instead, is associated (or identified) with the axiomatically-based neowalrasian analysis (Clower, 1995). This juxtaposition, revealing Clower’s own sympathies, is co-extensive with the distinction of analyses in terms of M-World or in terms of R-World models. While the former would be purely imaginary constructions, the latter could claim empirical foundations.

¹As cited by Clower from Hicks (1982), Collected Essays on Economic Theory, vol. II.

Borrowed from Singe², this distinction applies to all disciplines, economics being just one of them, and has to be handled with care.

About Axiomatics Clower's two main theses are. Axiomatics is useful in deductive or demonstrative disciplines; but it has no positive value in disciplines based upon plausible reasoning.³ Economics is of the latter kind. If axiomatics has any use at all there, it is only when and if it used for a critical role, as contrasted to constructive role which could only be played in the former disciplines. Otherwise, it is likely to be of as "little use to an empirical scientist as a broken saw to a carpenter"(1995, p.310). These theses have a logical preamble. The difference between economics and the physical sciences is that "the subject matter of economics is the stuff of everyday experience" (Clower and Howitt , 1996, p. 21).

That economics should not be a deductive discipline, is a strong thesis which is obviously part of the argument to demonstrate. While I will leave it there for the moment, I just want to recall that the recent conversion into a strictly deductive discipline is part of a general restructuring of the sciences, hard and soft ones alike. Such development may have proved only temporarily successful, sometimes a total failure, as Clower himself points out (Clower, 1995, pp. 308-9). Nevertheless, deductivism coupled with the axiomatics method, has exerted an almost irresistible and generalised power of attraction in this century. It appeared as an all-purpose approach that could be extended beyond Mathematics, to realise methodological unification around a hard nucleus of mathematical certainty. Inductivism on the contrary seemed to be confined to some of the sciences. Clower's belief that there are two distinct sorts of disciplines is in question here.

Such understanding of deductivism is a new and fairly recent development of the debate on the Foundations of Mathematics.⁴ Economics got involved into a process started elsewhere, which generated the neowalrasian approach or "code" according to Clower and Howitt (1996); or else "research programme", as called by e.g. E.R.Weintraub (1985). The reasons for its overwhelming success and of its unchallenged dominance for some decades now, have to be explained, and I offer some general arguments in Section 2. Resistance and criticism of the neowalrasian approach have increased recently and dissatisfaction has been voiced by a number of people⁵. It does not seem to me that all this has actually undermined its position. I surmise that these attacks, whether they came from within or from without the supporters' ranks, have fundamentally missed the target.

Clower's critique of the neowalrasian approach and its Citadel, General Equilibrium Theory is an instance of external critique, which is founded on a learned reading of (the history of) logics and mathematics of this century. Its evaluation will offer occasion for reviewing some of Lakatos's work on the issues of the foundations and the philosophy of mathematics, which is perhaps less known compared to his Methodology of Scientific Research Programmes (MSRPs). This is the topic of Section 3. I argue that varieties of axiomatics have to be distinguished, and that there were (and there are still) available alternatives to the one so far dominating.

²From Syngé: J.L. Talking about relativity, (Amsterdam: North Holland), (1970);

extensively quoted in Clower (1995).

³ Inspired by G. Polya, "Mathematics and Plausible Reasoning", in Polya (1954),

Induction and Analogy in Mathematics, 2 vols., (Princeton: Princeton University Press).

⁴For a good history, see Boyer (1968).

⁵see Kirman (1987) for a discussion, and the references cited there.

I find, on the other hand, a little surprising that epistemology of economics be contrasted to that of the physical sciences (the preamble cited above). In my views, it is more appropriate to appraise economics in comparison to mathematics for the historical conditions of their development in this century, but also because mathematics provides the language of formal modelling in economics. One cannot deny that physical sciences, too, have to do with the stuff of everyday life.

I do agree with Clower's statement that Walras and Hicks' works are concerned with R- instead of M-models. I share the view that there is a profound gulf between the Classical treatment of General Equilibrium and GE Theory by the neowalrasian approach. To quote Clower again: "The aim of Hicks and the other precursors of the Arrow-Debreu theory (he means pure theory, here), specially Marshall and Walras, was to construct a formal model to interpret aspects of the experience that would reveal the "fumbling fingers", as it were, of the Adam Smith's invisible hand". Clower does not explain how they accomplished this but the contrast with Debreu is sharply put: "By contrast, the aim of modern contributors to the neowalrasian theory has been to invent worlds in which the invisible hand works perfectly" (Clower, 1996b, p. 52). Finally, I also agree that "What was in Debreu's mind is neither here nor there; from internal evidence in The Theory of Value I conclude that the world Debreu intended to model was his conception of Hicks' Value and Capital (Clower, 1995, p. 313). But how did Debreu manage to convert an R-world model into the canonical M-World model of the neowalrasian analysis, when "In the first case the aim is to correlate theory with experience; in the second, to correlate theory with preconceived ideas" (*ibidem*). This historical switch by itself shows that there are more uses to axiomatics. Section 4 reconstructs the divide between the two approaches to GE as a matter of different axiomatic structures. The final two sections draw the implications and suggest what will be, in my view, next century divide.

2. Taking the neowalrasian approach seriously

There is something in the neowalrasian approach that makes it sound like the chant of Sirens to the economists' ears. It is Clower himself to confess it in a nice and candid way. It may be true that "...in effect, the neowalrasian code acts like a black hole, consuming everything it touches and cloning even residual orts into an Arrow-Debreu monster", but "I (...) myself have felt the power of the neowalrasian code, and can now recognise (in the clear light of hindsight) how the code corrupts and weakens the scientific force of one's theoretical work" (Clower, 1995, p. 310). One should perhaps invent Ulysses wax earplugs. This power of attraction is the third thing to explain.

The issues raised above are obviously intimately related, but I will take them up one by one for clarity. I will begin with the story of the "fatal attraction".

The trouble with the by now many opponents of the neowalrasian approach is that often they do not take it sufficiently seriously. With some despise, they think of its core, GE, as a pure theory that can easily shown to be void of empirical significance, hence lying outside economics, which is presumed to be an applied science, and belonging to mathematical logic. To slip into this sort of conclusion, is a temptation in Clower's argument too⁶. Essentially the same view is entertained by the supporters of neowalrasian approach, it is only the value judgement that is different.

⁶ Perhaps with a little stretching, this might be a risk involved in a statement like this: "I

have the impression that Arrow-Debreu theory's" impact "has been the consequence of the position (...) as an authority symbol"'". (Clower, 1995, p. 317).

I think the trouble is twofold. First, axiomatics is a method or an approach used also in (many) other sciences, beside mathematics, which have learnt to live with it. Moreover, this kind of outwardly external criticism ends up leaving to axiomatics more room than it needs to have, and lends a sort of unintended support to its cause⁷. The conjugated statement that the direct influence of the neowalrasian approach have been limited to the high theory of a few mathematically gifted theoretical economist, and that it descends from its being a mere authority symbol, reveals gross under-estimation. The influence is vastly ramified, as it was planned to be, and descends from its proposal of supplying the foundations and validation for economics. Moreover, in the neowalrasian paradigm, there is a unity though hierarchical between theoretical and applied research.

Here, I myself have fallen into a typical mistake, when speaking of the axiomatic method per se without further qualification. I claim that there is no such a thing as a unique axiomatic approach: Axiomatics has always to be qualified. When Clower, and I above, talk of axiomatics in the neowalrasian approach we mean Formalistic, Hilbert, Bourbaki style, axiomatics. I need not expand on this now (some notes infra). For the Formalist axiomatics as providing a “standard of rigour”, Debreu in Theory of Value, specially the Preface, is very explicit⁸. The mere fact that the neowalrasian foundations be referred to the philosophy of mathematics of one of the three fundamental schools of thought of this century should per se provide a very good reason to take it seriously. This story has been commented in so much literature, that one can take it to be common knowledge. The point is whether there is (or was) an escape route, apart from inductivism.

We have to confront the laws of Formalism, its succes-s, the reasons of its failure if of failure one can speak. It happened in formalised economics what has happened elsewhere. The Formalist doctrine has wiped out all other views, up to a certain point in time, and has exerted a long lasting influence upon mathematical thinking, including of those that did not accept Formalism. It was attractive for many reasons, the first is that it supplied an answer to the paradoxes that seemed to undermine mathematical certainty at the turn of the century. But this is not the story which is interesting to us, for it does not account for the power of attraction outside mathematics.

Formalism was announced as a fully fledged programme for dragging the axiomatic approach to its ultimate consequences. The revolution brought about by David Hilbert in the Mathematics (and imported into the other sciences during the 20s and 30s) embodied the grand project of the human mind to free itself from the sensory substrate and constraints. It encapsulates the ideal tension in the dream of Icarus, flying above the world, a dream that is very difficult to dispose of. After Goedel, Hilbert’s dream of founding a self-contained mathematics as the foundations for all sciences, appears as the last attempt

⁷“I find no flaws but as a foundation of an empirical science...”, (Clower, 1995, p. 317).

This seems to me to imply that the role of neowalrasian analysis goes beyond that of “saddling the economic profession with the neowalrasian code” (ibidem, p. 312).

⁸Actually there is a whole literature on this. There are various writings -by G. Debreu, specially after his Nobel Memorial Prize Award; Hildenbrand’s preface to Debreu’s collected writings; Ingrao and Israel (1990); Weintraub (1985); Mirowski and Weintraub (1994). For full references, see Clower (1995). I also have two papers on this issue, Punzo (1989), (1991).

of many. Goedel showed it to be a dream beyond human capacities, hence not exactly like Icarus' dream, for eventually mankind found a way (actually many ways) to fly. Although I do not sympathise, I think one should mourn on its disappointing end, as one cannot forget that that launching the assault to heaven is one of the dreams in the history of mankind.

The, at least, temporary domination of Formalism was the result, on the other hand, of the fear of the open world, the rejection of the plausibility and fallibility, for want of the absolute and the certainty. The version deployed in the neowalrasian approach is to a certain extent softer, Bourbakism entertains a relativistic view where mathematics is a set of structures (while hard-line Hilbert's philosophy introduces a hierarchy among them). But it still suggests the same message in promoting the conception of economics as a set of models⁹.

A second look at an alternative is forced upon us by the works of Goedel and others, some very recent in Artificial Intelligence literature. Inductivism may be one alternative, but perhaps there are others.

Although the case for a limited and "regulated" use of axiomatics finds me sympathetic, I plan here to defend the cause of axiomatics. I do agree that the problem with Formalist axiomatics is not lack of realism, but that "it inhibits coherent intellectual analysis, that is serious theorising, about observable events." This is not true elsewhere, however, so that the issue at stake is not a bold contraposition of axiomatics against non axiomatics¹⁰, which may turn to be (one of) the wrong alternative(s) that have presented themselves so often in recent years. There is more than one axiomatic system, and there are various ways of doing axiomatic to be explored in mathematics as well as in economics. We should exit from the twin identities of axiomatics being equal to neowalrasian approach, and of Formalist axiomatics be tantamount to rigour and clarity of theorems. We can accept axiomatics as a testing ground, produce alternative formulations, and discuss the "values of theorems" so obtained.

Axiomatics has a bad habit, that once it is let onto the stage, it seems to leave room for nothing else. The equilibrium advocated by Clower with some use for axiomatics in disciplines based on plausible arguments, is definitely unstable and difficult to discipline. To its users, axiomatics promises a freedom of thought un-heard of in any other approach. Axiomatics in the Formalist mode goes as far as to allow only for "self discipline".

3. (Garden) Varieties of axiomatics

In the last 30 years or so, economics has witnessed repeated methodological disputes, if not crises, punctually recorded in the literature. To mainstream economics, they seem the reflection of the growth process through which economics has gone, and still is going. To the assorted bunch of critical economists to which Clower belongs, their near cyclical recurrence manifests the uneasiness of working in a discipline progressively losing contact with reality, something codified and rationalised with the separation of theory as applied investigation from pure theory.

I suggest that a situation similar to the one in mathematics has arisen in economics since the birth of the neowalrasian and its full articulation. "It frequently happens in the history of thought when a

⁹See Mirowski and Weintraub (1994); Punzo (1991).

¹⁰Nevertheless, I am inclined to accept Clower's statement that there is "no way to make progress in economic science except by first discarding the neowalrasian analysis" (1994, p. 810).

powerful new method emerges the study of those problems which can be dealt with by the new method advances rapidly and attracts the limelight, while the rest tends to be ignored or even forgotten, its study despised. This situation seems to have arisen in our century in the Philosophy of Mathematics as a result of the dynamic development of metamathematics. (my italics)” (Lakatos, 1976, p.I)

While Metamathematics is backbone of the Formalist programme (its “core” in Lakatos’s terminology), likewise General Equilibrium theory is the backbone of the neowalrasian programme¹¹. By the neowalrasian supporters club, it is proposed as the Meta-theory for economics, a Metaeconomics, a term coined by Karl Menger at the birth of the neowalrasian approach (K. Menger, 1936, 1979). Much of Lakatos’s characterisation of mathematical Formalism can be easily paraphrased and adapted to Metaeconomics. For instance, “I shall refer to the school of mathematical philosophy which tends to identify mathematics with its formal axiomatic abstraction (and the philosophy of mathematics with metamathematics) as the ‘Formalist’ school.” This characterise the neowalrasian brand of General Equilibrium Theory. Then, “The subject matter of metamathematics is an abstraction of mathematics in which mathematical theories are replaced by formal systems, proofs by certain sequences of well-formed formulae, definitions by ‘abbreviatory devices’ which are ‘theoretically dispensable’ but ‘typographically convenient’.” (Lakatos , *ibidem*). This depicts the technology of model building in the neowalrasian mode.

Mathematical Formalism’s two fundamental tenets are that axiomatics is the unique method to ensure rigour and intellectual discipline, and there is only one way of “doing axiomatics”. They imply that disciplines cannot be Manicheally divided between those producing R-world versus those producing M-world models, or between demonstrative disciplines and disciplines which would be based upon plausible reasoning¹². To Formalism, there is only one kind of discipline.

Then the question: is axiomatics useful (Clower, 1995, p. 307) must be reformulated in the more carefully worded: What sort of axiomatics can be useful, when and how, to challenge the claim to exclusive propriety over axiomatics by the neowalrasian program and to exit from an ill-posed choice between inductivism and deductivism.

Axiomatics as a method to derive propositions from some codified way of stating postulates, axioms and the like, defining the universe of discourse precisely, in other words identifying its borderlines or frontiers. Given a generic definition of this sort, the problem is to consider its practical implementations. These have such an unexpectedly rich history as to make the very concept of axiomatics a theoretical abstraction, and they exhibit a surprising variety that calls for a guidebook.

In his writings on the philosophy of mathematics Lakatos attempted a provocatory classification.¹³ According to it, there is no distinction between demonstrative and plausible discourse, for even the premier demonstrative discourse (mathematics and logics) does, or at least may, have some empirical content. All disciplines do have to face the problem of truth as an external check to the usefulness of the theories produced¹⁴. Following this interpretation, different “varieties of axiomatics” can be distinguished on the basis of the level of the truth injection.

¹¹ Punzo (1991).

¹² Polya, quoted by Clower from Polya (1954).

¹³ Mostly in Lakatos (1976).

¹⁴ As demonstrated by the studies of Lakatos on the renaissance of empiricism in mathematics, collected in Lakatos (1978).

The discriminating criterion is whether truth injection is at the beginning, that is at the level of hypothesis, during the building of the formal model; or at the bottom, after it has been built, when it is tested (if ever) against reality. These produce two canonical versions of axiomatics: “Euclidean” versus “Empiricist” the two of its conceivable realisations that historically have played a crucial role.

Neither versions rejects confronting with (“empirical”) reality, at least as far as the applied sciences are concerned, so this is not part of the dispute with inductivism. In the canonical, however, the test for truth and discipline is not built into the modelling procedure, hence results (that is theorems) derived via logical procedures may or may not correspond to specific actual realities or be confirmed/ falsified by experiments. This may happen now, in the past or in the future, or never. Starting from this the neowalrasian programme may produce, and in fact it did produce, its own brand “applied economics”, proving that its direct influence has been much deeper than often conceded.

Lakatos supports the thesis that Axiomatic Programmes have been devised to fight scientific scepticism and its weapon, infinite regress. Accordingly, the Euclidean programme is the deductive programme whereby truth flows downwards from axioms and postulates (“they are assumed to be true”, or “un-demonstrable”) to the statements or theorems obtained from them.

Some translation work is needed of the considerations from the philosophers of science to economics, if we want to understand their bearing on our debates. They argue that the crucial distinction becomes here the one between two versions of the Euclidean programme, the Formalist and the Intuitionist version that dominated before Formalism took over. Although the latter must be counted as an entire orchard of axiomatic varieties, I call it collectively Intuitionist to retain the parallel with this century’s history of mathematics and the debate on Foundations. In contrast to the Formalist view, here primitive terms are not merely symbols in a game whose meanings are bestowed upon them by their use in functional relations. They are founded upon a pre-theoretical intuition and refer to actual realities (phenomena, processes, states) that are represented in the formal structure of models. Intuitionists plan to build models of the world (of the R-world in terminology used above), or to model “phenomena of the actual world”. Hence, Euclid geometry claimed to be the “natural” geometry of the space and time as we perceive it, and this is the “truth” that would be flowing according to Lakatos downward the deductive chain¹⁵. This was also taken to be the justification for its undisputed dominance over more than two millennia (at least until Gauss and a bunch of Russians met with unthought-of logical possibilities, at the beginning of last century).

I understand that the brand of deductivistic reasoning (with its own axiomatics) advocated by Clower is what I called Intuitionist. Intuition is the ultimate source of useful axiomatics, sometimes an intuition experimentally founded as “...every empirical science provides ample material to construct intellectually provocative models of myriad M-Worlds. Such models, whether in sociology, geology, physics, economics, meteorology, ecology, or evolutionary genetics, must be judged primarily on the basis of logical consistency and only secondarily on the basis of consilience with R-Worlds evidence.” (Clower, 1995, p. 309)¹⁶.

¹⁵Whig history of the neowalrasian approach sees no alternative to Formalism.

¹⁶“Bearing in mind that “reality” never matches what we “cave prisoners” might judge it to be, any other attitude would be absurd.(...) More to the point, a good theory can never be harmed and may be helped by normalisation; so by and large I favor axiomatics in disciplines rooted in plausible (inductive) as well as demonstrative (deductive) reasoning.”

This “intuition” anchor is all-pervasive in the unique descriptive construction of Intuitionistic axiomatics. Its absence, or its lack, induces the undistinguished plurality of models of Formalist axiomatics. The history of the latter proceeds from the discovery of the logical possibility of multiple geometries. Here, each “model” is the “general theory” of a virtual R-world but obviously it cannot be validated by any (experimental) correspondence. Models are played with to “see what would be the consequences in a “world” where model assumption(s) could be taken to be true”. Hence, Pure Theory generates as a catalogue of “model economies”.

Validation of the single conceptual possibilities envisaged on the Formalist catalogue (and of the cataloguing itself as a “scientific” procedure) must come from complying with the dicta of a higher level theory, a Metatheory which must be created on purpose to provide missing foundations. Here is one of the Hicksian rabbits, which seem to jump out of many people’s hats, and not just from those of the economists. “Scientific” magic is a popular divertissement, even among followers of rigour and discipline as taught in Formalist axiomatics. The fragmentation of the Formalist theoretical world, relativist, but true, matches the provocative richness of thought experiments of Intuitionism.

The relevance of the above articulation within the Euclidean programme is best illustrated by a comparison of Walrasian and neowalrasian versions of the general equilibrium analysis. This theme has been the subject of much research recently in a historical vein greatly inspired, favourably or not, by the other Lakatos, associated with MSRPs and the research hypothesis of Rational Reconstructions. Though with a variety of nuances, all entertain the notion that GE theory is at the heart of the neowalrasian approach; those accepting in one form or another the jargon of Lakatos’s MSRPs, would maintain explicitly that GETh is the core of the neowalrasian Programme (a classical reference for this thesis is Weintraub (1979), (1985))¹⁷. As for our argument, positions range from that represented by Weintraub himself (as the leading representative of the “discontinuistic” interpretation of the relation

(Clower, 1995, p. 309; my italics). The question of how to spot a good theory is obviously

left open. Lakatos has ironical comments on this way out.

¹⁷See Weintraub’s definition of the hard core in Weintraub (1985).

The qualification in the text is appropriate, where it is suggested that the core might contain other things beside the GETh. For this position, see Backhouse, who seems to partly accept the MSRPs and partly criticises the distinction between the core (whose definition would be too narrow) and the protective belt. See below, for further comments on Backhouse.

between the two), with Mirowski and Weintraub (1994) on the other extreme, while Ingrao and Israel (1990) occupy a sort of intermediate position.¹⁸

The main issue being disputed is the status of the neowalrasian approach. It is time to get on this track.

4. Temptation(s) and sin. Or: where do all these rabbits come from?

One is taught to speak of General Equilibrium Theory tout court (the temptation) and to re-read what happened in this area in the last 100 years through the eyes of the partisan history of Arrow and Debreu¹⁹ (the sin). This should be no surprise. Formalism does not allow for history (Lakatos, 1976 p.I). Therefore, its “history” of GE only sees its own ancestors and precursors. This view is another example of a sirens’ chant economists seem to be so prone to listen to, which explains its great popularity. In recent years, it has been dressed up as Latakos’s MSRPs (and will be exemplified hereafter by Ingrao and Israel, 1990).²⁰ In contrast, I will argue that there was an axiomatic alternative to Formalism, which would have preserved the intuitive foundations of Classical General Equilibrium analysis. This possibility is invariably missed in the literature.

The standard reconstruction of the development of General Equilibrium analysis is construed around the thesis of the presence of a basic structure, a “canonical model”, built around a historically “invariant paradigmatic nucleus”. I pick up the wording and argument put forward by Ingrao and Israel (1990) as a good illustration of this modern interpretation. They argue that such nucleus contained the analytical issues of proving existence of an equilibrium, its stability and global uniqueness. In Clower’s words, there would be the same M-world model throughout the history of the subject²¹.

¹⁸Discontinuistic means that to Weintraub the neowalrasian approach represents a new Research Programme in the sense of Lakatos, compared to the Walrasian approach.

Mirowski maintains that economics has fallen prey of the physics envy in the eighteen century and never got over it, whence a continuistic thesis. Ingrao and Israel are rather ambiguous about this,, marrying one side or the other in different contexts.

¹⁹There are too many versions of it to single out one of them, but, for historical record, see introduction to Arrow and Hahn (1971) and Debreu’s Introduction (1959).

²⁰ Economists have flirted with Lakatos of the Methodology of Scientific Research Programmes at least for some time in the eighties. Backhouse (1991) asks the reasons for the wave of criticisms that have been volleyed against it soon afterwards. I would rather inquire about the reasons for that initial, and apparently superficial, success among our profession.

²¹Clower, obviously, would not agree with Ingrao and Israel’s reconstruction, as appears clearly on p. 313 of his repeatedly quoted Clower, (1995).

This sophisticated version of whig history does not allow any fundamental difference between old or Classical GE analysis and the modern Arrow-Debreu version, and to critics it well supplies further evidence (if there was any need) of the subtle fascination of the neowalrasian code. Nevertheless, as it proved unable to demonstrate the properties of stability and uniqueness of equilibrium, serious doubts about the internal consistency of that “canonical model” are advanced and GE Theory is said to have reached a dead end.

Such strong conclusion rests upon the possibility of identifying the postulated basic structure common to walrasian and neowalrasian models. The existence of such a thing as an invariant nucleus is a false problem, both in Clower’s and in my views. The basic structure should be obtained as an intersection. That the latter is empty in this case, can be best illustrated by formulating in a explicit axiomatic form the structure of the Walrasian model, which was not put into axiomatic form. This requires formalisation according to certain simple rules, or a translation into an axiomatic language, so as to make it comparable with the formulation, already axiomatic, of the neowalrasian model. This use of axiomatics, for clarification and critical comparison, is very much an exercise in the vein suggested in Clower (1995).

I will not go through the whole exercise here (as I have done it elsewhere)²²: It would fully illustrate the peculiar way in which axiomatization is done by a Formalist economist, compared to what I call an Intuitionist treatment of General Equilibrium. I will simply extract the central point, focusing on the central and discriminatory role assigned to “intuition”. In this light, the distinctive characteristics of the Classical GETh emerge neatly if one takes as an assumption that its GE model be restricted to the subspace of scarce goods. Although the relevance of this assumption is never fully appreciated, historically the radical novelty that brought about the neowalrasian approach was precisely its removal, with the prescription that the space of economic goods be always determined endogenously. GE analysis became “general” as understood in the modern sense as the result of the formal enlargement of the universe of the data, to encompass pre-economic (or not yet economic) data.²³ While in Classical GE the goods of the theory are “everyday commodities”, intuitively defined, in the neowalrasian they become pre-economic entities. As they are defined within the network of formal relations in which they are used, they can be anything. As no theory-external content is assigned to the terms used in the

²² Punzo (1989), (1991).

²³I am referring here to the original simplified model discussed in Vienna and finally solved by Wald. This focused on the issue of proving the existence of a general equilibrium in a setting where goods could become free, but factors could not nor technology could be chosen and tastes were disguised behind the given demand functions, à la Cassel. This model has been discussed at length by Weintraub (1979), (1985). In Punzo (1989), I reconstruct the logical history of the so called rule of free goods, and its role in the process of shaping up of the Formalist approach to economics, i.e. K. Menger’s Metaeconomics.

language of the theory, obviously there is no project of “constructing an empirical science in which the relationships studied are matters of everyday experience” (Clower and Howitt, 1996, p. 34).²⁴

The principle according to which model data had to be treated as pure symbols, was later extended to the whole set of fundamentals, first technology and then tastes. The neowalrasian approach used the existence issue not to make economists discover axiomatics, rather to change the axiomatics they were familiar with.²⁵

Severing the link with basic intuition implies that the model as a description is replaced by a formal system in the sense of logic. If the subspace of commodities proves to be empty, and all goods are free, the equilibrium is trivial, the associated model meaningless. The non existence of a (non trivial) equilibrium can only indicate an internal logical inconsistency. “Proving existence” becomes proving consistency within a formal system. This is however not the only equilibrium notion that can be entertained.

In mechanics, an equilibrium is a rest point of a moving system or a stationary state of a process. If an economic equilibrium is assumed to be known, with all trades realised, the list of priced goods is also known. Hence, to make sense of the classical treatment of GE, we should think of it as working with a local model in a neighbourhood of an equilibrium state. In the geometric terminology of dynamical systems, a local model is a dynamical system on a restriction of the state space to a neighbourhood of some relevant state. The Classical approach develops within the borders of the surrounding sub space of scarce goods.

The problem is to choose the equilibrium to start with. The classical approach does this by starting from a sort of primitive intuition of a system given state and by boldly assuming it to be an equilibrium. I call models of this brand, based upon a pre-theoretical i.e. theory-external interpretation of data and on realized data, “state space models”. Classical GE models span this class, and this is where they exhibit their mechanical analogy. The neowalrasian canonical model, by contrast, is a global model built on “raw data”. Goods, agents, institutions are treated as primitive un-interpreted terms; markets are possibilities of relationships that may or may not materialize. Hence, the “model economy”, which \.may or may not come to life. This is the way some of the Hicks’ rabbits get into the hat. When they come out, we need to know about existence, stability and uniqueness (or finiteness of the set of) equilibria.

5. Exploring theoretical modes

In economics, I maintain, there are at least two modes of handling the “axiomatic method”, even if we do not question deductivism. We need two distinct axiomatic structures to make sense of the two existing canonical versions of GE Theory and to see where and why they differ. They are associated with different levels of truth injection. In the classical mode, commodities recall “images of the stuff of everyday life” (truth is injected at the top). In the neowalrasian mode, they have to be determined or

²⁴That they were very conscious of making a big leap forward, is clear from the words of Karl Menger, see in particular Menger (1973).

²⁵What sort of mathematics was familiar to economists, is itself an interesting research topic. K. Menger (1973) points out that, in Vienna at least, they were not as innocent of mathematics as we are told by Arrow and Debreu.

better “discovered”, and may or may not coincide with the commodities that we know (and as we know them). Truth injection is “possible” but “not necessary”. If it comes, it is a blessing from a theorem or a corollary at the end of a logical chain of arguments. Truth injected at the bottom. In this rigidly regulated game, familiar names of economic reality are mere “interpretations” in the language of everyday life (Debreu, 1959, Preface). To paraphrase Lakatos, economics is identified with its formal axiomatic abstraction. Interpretations are values that may be assigned to local variables. Value assignment belongs to Theory instead of Pure Theory, located at a lower level or later stage in the theoretical process. One should not marvel at the state of monetary theory in the current mainstream, because “money is not there” or it is there in virtue of a variety of ad hoc constructions.²⁶ This results from the absence of all ¹commodities and institutions as we know them, that comes from the programmatic rejection of intuitive foundations. Working with primitive terms may lead to contradictions to intuition and experience, that can only be prevented by adhockeries constraining theoretical outcomes. As Clower’s work shows, current theories of money and markets offer the most revealing case study.

The change in the notion of equilibrium has removed the analysis of GE from the realm of dynamical systems shifting it to that of logic and set theory.²⁷ While it used to be the co-ordinate description of a system state, equilibrium has been reduced to an internal test for model acceptability. From this derives the intrinsic static flavour that prevents telling a reasonable story about it, explaining and teaching equilibrium as the outcome of an actual process. This produces also the separation between statics and dynamics and transforms dynamic stability into the property of an optional “associable” dynamical systems.²⁸ The stability issue, as it comes out in the neowalrasian approach, is a typical example of a Hicksian rabbit: it is a separate issue because it arises from the way theory is being constructed.²⁹ There is something else to explore.

In my argument above, the classical GE model is constructed as a local chart of the economic world. General Equilibrium in the neowalrasian approach supplies the cartographic technique to produce a sort of Atlas, for all times and all conceivable worlds. Formalism of modern GE Theory has taught us to be over-ambitious, promising results that have not come out and indicated a in-human task.

²⁶See Clower and Howitt (1996) for perceptive criticism of the current approaches and of the adhockeries in monetary theory. Such criticism represents the link between Clower’s monetary theory and his views on methodology.

²⁷ Debreu (1959). See also von Neumann and Morgenstern (1944). The “meretriciousness of the economists’s notion of equilibrium” (Clower, 1995, p. 317) only shows one of the many facets of the adhesion of economists to Formalism: their fundamental schizophrenia.

²⁸For a criticism of this conceptual separation codified by Samuelson’s Foundations, and of the current interpretation of dynamic stability, it is useful to read the postscript to the second edition of Value and Capital (1939).

²⁹In other words, “because of” the neowalrasian mode. This refutes Ingrao and Israel thesis of the existence of a time-invariant analytical nucleus in the history of the subject.

For want of an Atlas, collections of fantastic and deformed maps appear (just like those produced in the Medieval times) and then we try to find out if there is a chart that fits best to each case. On the basis of this Atlas people venture beyond the Pillars of Hercules, into applied economics and economic policy. This has nothing to do with the Pygmalion Syndrome Clower recalls. The neowalrasian economist, just like the formalist mathematician³⁰, lives a schizophrenic life, full of grand ambitions that have to make do with the experience of everyday life. Schizophrenia descends from accepting the need for a Metatheory and the preaching of Formalist Metatheory. Behind the “structures of economics”, there is a common Metatheory, which is exemplified by its General Equilibrium core. This circumscribes the walls of the Citadel, but it does not fill it up. The Citadel is fully manned by a variety of non (necessarily) descriptive “models”, one of them being the General Competitive Equilibrium model. Almost always, external and internal criticisms are brought against the latter, a target easy to knock down leaving the walls untouched.

One cannot avoid the feeling that really the neowalrasian approach is after some sort of God’s Economics, just like there was such a thing as God’s Mathematics that we “should let God do by itself”. It is a lifeless, institutionless, money-free economics. The clock of history is broken at time zero. We always start from the beginning of history and decide everything for eternity. There was no past, there will be no interesting future. It is this teaching of Formalism, the absence of history, past but also present and partly future, that some economists and an increasing number of scientists everywhere find difficult to accept. They take for a failure the lifeliness of formalised economics³¹ which neowalrasian economists praise as its principal and most desirable virtue.

Are there alternatives? There are other modes of doing mathematics. Accordingly, there are other ways of doing modelling in economics. We have to reject the conceptual trap implied by the juxtaposition of Pure Theory, which is not a Theory but the set of instructions of a content-free Metatheory, and Applied Theory, which is the set of interpretations. There is no complementarity nor division of labour between them; the distinction implies a hierarchy. We should revert to Theoretical Economics, “considered as the logical analysis of an economic system (of free enterprise)”, (Hicks, 1939, p. 7), a wording that well exemplifies in the clearest way the Intuitionist brand of the Euclidean Programme in economics.³²

Treating a model as a navigational aid for the small world surrounding the observing economist, or else as a conceptual laboratory for a phenomenon, instead of the description of “a conceivable world”, looks less honourable to the mainstream neowalrasians. What we get, in exchange, is one plausible, though partial, picture of parts of the actual world instead of a whole bunch of them from which to

³⁰This is the same psychological disorder, schizophrenia, of which are said to suffer the mathematicians interviewed in Davis and Hersh (1981): to paraphrase their argument, they would tend to be formalist when they “think mathematically”, but intuitionist in everyday life, when they look for meaning and interpretation

³¹Leijonhufvud: “Acceptance of the neowalrasian code has taken all life away of economic theory” (this volume , p.??). I am suggesting that this was intentional.

³²Notice that it was written before the advent of the neowalrasian approach, which Hicks opposed.

choose without a criterion to select. Practically, all issues that have been emerged in recent literature critical of the dominant mainstream (e.g. from the discussions on bounded and procedural versus objective rationality, learning and expectations, aggregation problems, and many more) revolve around choosing between a strategy of working with a local model, or many local models of system behaviours, and with a unique global but “improbable” Atlas populated of fantastic countries. Here is the -origin of the dispute.

It is this century’s divide. We should accept and build on this principle of locality.

5. This (and next) century’s divide?

I have already stated my main contention. To understand what is going on in our discipline, we must start from the plain observation that there is no such a thing as the axiomatic method. The close association between axiomatic (re-)formulation Hilbert-Bourbaki style deductivism with the neowalrasian approach is a historical accident in the evolution of theoretical economics. There is still room for choice, as there was choice already at the time the accident took place.

My answer to Clower’s question is that Debreu did not intend to help the development of economics as an inductive discipline.³³ However the issue at stake in Theoretical Economics seems to me not just inductivism against deductivism. It is also a matter of choosing between intuitively-founded local models and global logically-founded models. The XIX century discovery of the new Geometries did not abolish the time honoured Euclidean geometry, it made us realise its relativity.

This “principle of locality” and therefore pluralism is embedded into the philosophy of Computable Economics. It is also the greatest common divisor for the variety of approaches that sprung up from the observation that mainstream’s economics is full of es camotages geared to avoiding the complexity of economic systems. They keep popping out of all the folds of the thick carpet under which they had been hidden. Even among representatives of the neowalrasian approach things are moving: so Hahn, “It so happens that it is becoming ever more clear that almost none (of the crucial questions facing the discipline) can be answered by the old procedures. Instead of theorems we shall need simulations, instead of simple transparent axioms, there looms the likelihood of psychological, sociological and historical postulates.” (Hahn, 1991, p. 47)

For once, however, I like to play moderate and to promote a suitably adjusted version of K. Menger’s Principle of Tolerance for mathematics.³⁴ I wonder if under the thick carpet laid by Formalist Axiomatics, there is still something to be learned. I would not be surprised if there was something interesting, though it may not be what we are looking for. Reasoning from axioms confines the mind to closed worlds. Inductivism opens it up, here is its superiority (if it is superior), and its intellectual appeal. Intuitionist axiomatics tries to keep some doors open, but one needs a lot of self-imposed discipline to avoid the “Euclidean curse” of ending up into axiomatics for its own sake. Thinking of Theoretical Economics as a discipline somewhere in the limelight between other disciplines, with

³³“...on balance, has Debreu’s Theory of Value helped or hindered the development of economics as an inductive science? Merely to pose this question is, according to some, tantamount to disparagement.”(Clower, 1995, p. 312).

³⁴ In Menger (1979), originally published in German in 1936.

thought experiments providing its fundamental analytical tools, supplies a reasonably good way to escape from the eternal inductivism/deductivism dilemma.

In essence, Clower's methodological stand on axiomatics and on deductivism questions the claim that it be possible to produce useful economics via a set of self-contained formal structures. One should focus on this and avoid to get trapped into pseudo alternatives and ill posed problems. Clower's methodological message, rephrased in my language, says that we have to come to accept the idea that the universe of economic experience is expandable, and is expanding fast, largely as a result of our own doings.

I think that the above exercise answers also the question why Formalistic axiomatics won over other approaches and it still dominates. It is because it is not a theory, hence it cannot be accused of not being that. It is a technique of reasoning, which is demonstrated by applied models. We may call a "code" what I prefer calling a Metatheory, that is the set of rules for doing theory in the neowalrasian mode, but we must make sure not to overlook its implicit message nor to under-estimate the strength of this subtle appeal. Neowalrasians themselves are happy with the code story.

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