FIRST BEST RULES IN A SECOND BEST WORLD: A GENERAL FORMULATION

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Abstract

We propose a general framework to analyze conditions allowing first best efficiency rules to be a valid guidance for policy in a second best world where redistribution must be implemented through distortionary taxes. This allows us to prove some well known results in a way which is remarkably simpler and more direct than what can be found in the public economics literature.

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1. Introduction

It is a tenet of the literature on optimal taxation that conclusions on policy issues such as the desirability of commodity taxes, the efficient provision of a public good or the optimal level of a pigouvian tax must be amended as we pass from a first best world where individualized lump sum taxes are available to a second best world where only distortionary taxes can be used.

When individualized lump sum taxes are available, first best rules (e.g. a public good is provided according to Samuelson's rule, a pigouvian tax is set equal to the marginal effect of an externality, etc.) constitute a sound guidance to policy because lump sum taxes can be set so that the implementation of such rules brings about a Pareto improvement. Starting from any initial distribution, a set of lump sum tax/transfers can be found so that all redistributive effects of the policy can be "neutralized".

Although these same rules do not generally provide a well-grounded guidance where less perfect instruments of distribution must be used, there are some circumstances in which first best rules carry over to a second best world. A well known example is the result that, in the Mirrlees framework where individuals differ only in their ability to earn income, a distortion of relative prices through differentiated commodity taxes is not called for when an optimal nonlinear tax is in place and utility is separable in commodities and leisure/labor (Atkinson and Stiglitz, 1976). Similarly, Christiansen (1981) proves that under the same conditions (weak separability and optimal nonlinear income tax) the rule for the optimal provision of a public good is just Samuelson's first best rule. Kaplow (1996) goes one step further: while advocating the use of the first best rule as the benchmark for decisions on public goods even in second best, he provides a simple way to deal with this and related problems, and shows how the result does not require that the income tax is optimal in any sense [1].

At the heart of Kaplow's argument is the observation that, when individuals have identical and weakly separable preferences, a suitable change of the income tax schedule is sufficient to neutralize the redistributive effects the provision of a public good may have on individuals. At each income level, and whatever the initial distribution and the existing income tax, it is possible to increase the latter by an amount exactly equal to the benefit that the individual at that income level receives from the public good. The change in total tax revenue corresponds to the sum of the benefits individuals obtain from the public good; if this exceeds costs—which will be the case whenever Samuelson's condition holds—then it is possible to make public good provision Pareto improving.

In this paper we develop a formal restatement of this kind of argument. This general formulation of the optimization problem, developed in section 2, allows us to emphasize the parallel between the first and the second best cases. In section 3 the formulation is specialized to deal with policy issues ranging from the provision of public goods and the correction of externalities to the desirability of commodity taxation. The generality of the formulation allows us to point at the common structure behind a variety of results, which can in fact be considered as special cases of a more general result on the possibility to

[1] See also Kaplow (2004b) for a similar result with regard to commodity taxation. As acknowledged by Kaplow, the claim that under these conditions public programs should not depend on distributional objectives, and only efficiency should count, can be traced back to Hylland and Zeckhauser (1979). For a general discussion on the possibility to focus only on efficiency when dealing with public policy, see Kaplow (2004a).

separate efficiency and distributive considerations in economic policy.

Although results close to the ones we present are already known to optimal taxation scholars, we are able to restate them in a way which is remarkably simpler than what is usually found in the literature. Contrary to the standard approach, our proof does not require the solution of a problem of optimization of the overall tax structure; indeed, as already emphasized, the results hold regardless of the optimality of the tax instruments used.

It is worth noting that the results are proved in a setting where a "mild" degree of heterogeneity among agents is assumed. Namely, individuals are allowed to differ in how they trade-off leisure and consumption; this makes our results somehow more general than the ones referred to in the literature, where attention is usually confined to the more restrictive (and unnecessary to our purpose) case of identical preferences[2].

2. General formulation

2.1. First best

We first characterize efficiency in first best, i.e. when individual characteristics are observable to the government, so that personalized lump sum taxes and transfers are feasible.

Let $v^i(k,z)$ be the indirect utility function of the individual i, where k is a variable describing the government's policy, and z is (net-of-taxes) income. In a first best economy, any socially optimal policy is Pareto efficient; it solves

$$\max_{k,z} v^1(k,z^1) \quad \text{s.t.} \quad \begin{array}{l} \sum_i z^i + c(k) = 0 \\ v^i(k,z^i) = \overline{u}^i \quad i = 2,\ldots \end{array} \tag{1}$$

where $c(k)$ is the cost of the reform to the government, and incomes have been normalized so that $\sum_i z^i = 0$ when $c(k) = 0$. With reference to the dual of the above maximization problem, we have that the optimal policy k_0 solves

$$\min_k \sum_i e^i(k, v^i(k_0, Z^i)) + c(k). \tag{2}$$

where e^i is the expenditure function for individual i, and Z^i represents her income in the optimum of problem (1).

2.2. Second best

We now rule out the possibility of personalized lump sum taxes, by assuming that the government has incomplete information about the characteristics of individuals.

Individual i's preferences are described by the utility function

$$u^i(m(x,k),\ell) \tag{3}$$

where x is a vector of consumption goods, k is government's policy (k can affect directly utility, e.g. k is the quantity of a public good), and ℓ is work effort. The sub-utility function

[2]Blomquist and Christiansen (2003) are a notable exception. They advance a taxonomy of cases of preference heterogeneity, and prove that, when individuals differ only in their preference for leisure, Atkinson and Stiglitz's result on the redundancy of differentiated commodity taxation still applies. See below section 3.3 on this.

$m(x,k)$ is assumed to be *identical* across individuals, although individuals may differ in how they trade off m and ℓ.

Let w be “productivity” , i.e. an unobservable vector of characteristics which affects the individual’s ability to transform “effort” ℓ into income. Gross income $y = y(w,\ell)$ is perfectly observable, while w and ℓ cannot be observed. With no loss of generality, we make the common assumption that $y = w\ell$. The government can levy a nonlinear income tax, and $z = z(y)$ represents net-of-tax income. Individuals may differ in their productivity, hence we write w^i to denote individual i’s productivity.

Each individual selects x in $B(k,z) \subseteq \Re^n_+$; the choice set $B(k,z)$ is a function of net income z and can be affected by the policy k (e.g. in the case the policy affects prices), but it is not affected independently by w or ℓ. Let $\phi(k,z) = \max_{x\in B(k,z)} m(x,k)$ be the indirect sub-utility function.

The optimal choice of ℓ by individual i solves

$$\max_{\ell} u^i(\phi(k,z(w^i\ell)),\ell). \tag{4}$$

Let k_0 denote the initial public policy and let k be an alternative. Then $\mu(k,k_0,z)$, defined by

$$\phi(\mu(k,k_0,z),k) \equiv \phi(k_0,z); \tag{5}$$

is the *(indirect) compensation function* for an individual whose initial net income is z. (We have $\mu(z,k,k) = z$ for all k.) Knowing the utility function ϕ, the government can calculate μ, because the effect of the change in k for individual i depends only on i’s net income; this would not be the case if B or m (and therefore ϕ) depended directly on w or ℓ, since these variables cannot be observed by the government. Note that compensation *does not* depend on the utility function u^i or the individual productivity w^i: it is the same across individuals.

If the government accompanies the “reform” $k_0 \to k$ with a tax change from $z(.)$ to $\mu(k,k_0,z(.))$ then, independently of the selected k, the maximization problem for type-w individual is

$$\max_{\ell} u^i(\phi(k_0,z(w\ell)),\ell) \tag{6}$$

given identity (5). Hence for all i the optimal choice of ℓ and the corresponding utility level are not affected by the reform.

Let Z^i be the level of pre-reform net income selected by individual i; the income tax of this individual is reduced by $\mu(k,k_0,Z^i) - Z^i$, which is equal to her compensating variation for the reform. The reform $k_0 \to k$ plus the tax change leaves all individuals at the initial utility level, while bringing about a change in tax revenue. The additional revenue accruing to the government, net of the cost $c(k) - c(k_0)$ of implementing the reform, is

$$-\sum_i [\mu(k,k_0,Z^i) - Z^i] - [c(k) - c(k_0)]. \tag{7}$$

If (minus) the sum of compensating variations exceeds the cost of the reform, the government gets a surplus, which can be used to obtain a Pareto improvement (e.g. it could be distributed to the individual through uniform lump sum transfers).

To make a comparison with the condition for first best efficiency, and in order to address some specific policy problem, it is useful to derive a general formulation for second best efficiency in our setting. We can state the following

Proposition 1. *Given any income tax—hence any net-of-tax income function $z(.)$—a policy represented by k_0 is optimal relative to $z(.)$ only if* (7) *is non-positive for all k, i.e. only if k_0 solves*

$$\min_k \sum_i \mu(k, k_0, Z^i) + c(k). \tag{8}$$

□

Remarkably, condition (8) is equivalent to the one we had in the first best (see (2)). Let us summarize the conditions that make the efficient criterion for first and second best equivalent:

- utilities are weakly separable in leisure/effort;
- preferences for purchased goods x are identical (and identically affected by k) across individuals; this implies that individuals with the same net-of-tax income choose the same x;
- a nonlinear change of the income tax is feasible.

Note once again that we require neither that the income tax is optimal nor that individuals have the same preferences with regard to the choice of leisure vs. purchased goods.

3. Specialization of the result

We now show how the framework developed above can be specialized to deal with some well known results in the literature on optimal taxation in second best. This should make clear that these are all instances of the same result.

3.1. Public goods

We specialize the problem by letting k indicate the quantity of a public good. We have $B(k,z) = \{x | px \leq z\}$ and $c(k) = p_G k$ where p_G is the constant marginal cost of the public good. By assuming that the public good affects (only) m, we allow the benefit to vary with income and can be complementary/substitutable w.r.t. private goods, though we rule out complementarity/substitutability with leisure.

The first order condition for the minimization problem (8) is

$$\sum_i \frac{\phi_k}{\phi_z} = p_G \tag{9}$$

which is Samuelson's condition. This is the result proved by Christiansen (1981) and restated by Kaplow (1996).

3.2. Externalities

Suppose that individual utility depends on an externality E, so that $m(x, E)$. As in the case of the public good, we are considering the case that the effect of the externality does not depend directly on individual ability or on labor supply, though it can depend, and will depend in general, on consumption of other goods, hence on the income level of the individual. Our setting can include e.g. the case in which the externality is created by rich people and affects mainly poor people, or other cases in which it has a distributive dimension.

We assume (without loss of generality) that the externality is a function of X_1, the aggregate consumption of commodity 1. The government can affect this consumption choice: in the case in which regulation takes the form of a Pigouvian tax[3] k

$$B(k,z) = \{x | kx_1 + px \leq z\} \tag{10}$$

so that an individual with net income z chooses $x = x(z,k)$; the level of externality as a function of k is

$$X_1(k) = \sum_i x_1(Z^i, k). \tag{11}$$

We have $\phi(k,z) = m(x(z,k), X_1(k))$, and use Proposition 1 to find the optimal level of k. Considering that $\partial\mu/\partial k = -x_1$ and that $c(k) = -kX_1$, we have the following first order condition

$$\sum_i \left[x_1(Z^i, k) - \frac{m_E}{\phi_z} \frac{dX_1}{dk} \right] - X_1 - k\frac{dX_1}{dk} = 0 \tag{12}$$

or, simplifying,

$$-\sum_i \frac{m_E}{\phi_z} = k \tag{13}$$

so that the tax should be set equal to the (unweighted) sum of the the marginal effects of the externality. Neither concern for distribution nor the fact that income taxation is distortionary justifies the dismissal of the first best condition.

This is essentially the same conclusion of Cremer, Gahvari and Ladoux (1998) that the optimal tax on the externality generating good is strictly Pigouvian when individuals have identical marginal rates of substitution at any given consumption bundle[4].

3.3. *Commodity taxation*

In this case, $B(k,z) = \{x | (p+k)x \leq z\}$ with $k \in \Re^n_+$, and $m(x)$. We set $k_0 = 0$, and consider that[5]

$$Z^i = e(p, \phi(p, Z^i)) = ph(p, \phi(p, Z^i)) \tag{14}$$

$$\mu(0, k, Z^i) = e(p+k, \phi(p, Z^i)) = (p+k)h(p+k, \phi(p, Z^i)) \tag{15}$$

where e and h are respectively the expenditure function (referred to m alone) and the vector of compensated demands of individual i. We have $c(0) = 0$ and

$$c(k) = -k\sum_i h(p+k, \phi(p, Z^i)) \tag{16}$$

(the "cost" is in this case the revenue from commodity taxes). Thus, expression (7) can be written as

$$\sum_i [e(p, \phi(p, Z^i)) - ph(p+k, \phi(p, Z^i))] \tag{17}$$

[3] As an alternative, we could have examined the case of quantity regulation by considering $B(k,z) = \{x | px \leq z \text{ and } x_1 \leq k\}$.

[4] The possibility to apply the result to the case of externalities was mentioned by Kaplow (1996).

[5] Although we consider explicitly as part of the policy only commodity taxes, the utility function can be taken as implicitly taking into account the effect of some form of public expenditure, e.g. a public good, whose amount is fixed. This can be financed entirely by income taxation (when $k = 0$) or by a combination of income and commodity taxation.

By the definition of expenditure function, we have for each i

$$e(p,\phi(p,Z^i)) \leq ph(p+k,\phi(p,Z^i)) \tag{18}$$

so that expression (17) is always non-positive. Indeed, because of homogeneity of compensated demand functions, (18) becomes an equality—therefore (17) is zero—when k is proportional to p. In conclusion, under the stated conditions it is never optimal to have differentiated commodity taxation: we cannot increase welfare by distorting prices. This is the result in Atkinson and Stiglitz (1976), extended to the case in which the income tax is not necessarily optimal.

4. Concluding remarks

We have assumed that, conditional on their income, individuals have identical preferences over consumption goods. This assumption, which is indeed quite restrictive, implies that there is a unique increase in income for each income level which can sterilize the change in consumption induced by the policy (i.e. the change in k). This is the key to all the results we have discussed.

An advantage of the proposed approach is that, by focusing on the compensating change in the income tax, we have a better intuition of why and when the income tax can replicate the redistributive effects of other policies.

By the same logic, it is easy to understand why a straightforward extension of Proposition 1 to the case of stronger forms of heterogeneity among individuals or of non-separability of preference between effort and consumption is not possible. Consider the case that m differs across individuals, or that m is directly affected by w^i, so that it is not true anymore that individual with the same net-of-tax income make the same consumption choices: in this case, the reform will affect individuals differently even when they have the same income level, hence there is no way to find a change in the income tax compensating all of them at a time for the policy change.

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