Asset substitution and Executive Stock Option Evaluation

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Abstract

We evaluate executive stock options by modeling the asset substitution effect: the executive is allowed to control the asset price by choosing firm projects and therefore its volatility. The executive solves a stochastic control problem. We evaluate different type of stock option payoffs describing the optimal strategy and the value of the contract comparing it to what is obtained with traditional techniques.

Keywords: Executive Stock Options, Asset Substitution, Stochastic control.
Classification:

1 Introduction

The role of stock option plans in executive compensation grew up significantly in the last ten years. A large part of executives' compensation is now provided through stocks or stock option plans, a phenomenon particularly acute for new (risky) companies, such as new economy companies, see [13, 26]. Through options, executive compensation is linked to the company performance and therefore to the shareholder's wealth in such a way that classical agency problems between shareholders and managers pointed out in [21] result weakened. On the other side, being the option payoff similar to the one of a debt contract, this type of compensation induces an *asset substitution effect*: there is an incentive for the manager to adopt a risky policy, see [21]. This paper addresses the evaluation of executive stock options modeling the asset substitution effect.

A large literature is available on executive stock options evaluation, we recall the special issue of the *Journal of Financial Economics (2000)* as well as [6, 9, 25, 8, 26, 32]. Two main topics are addressed in the literature: evaluation of the stock option, analysis of the effects of this type of compensation on the management of the company by the executives. With few exceptions, these topics are addressed by using standard option pricing techniques, i.e., Black and Scholes style techniques. The analysis proceeds as follows: the executive stock option is described as a contingent claim contract written on the asset price which is modeled as a stochastic differential equation, then the value of the option is computed according to risk neutral pricing techniques. The effect of this type of compensation on the executives' behavior is evaluated by computing the derivative of the value of the contract with respect to the volatility, asset price, dividend ratio, etc.. Following classical option pricing results, the value of most executives' compensation plans is increasing in stock price and volatility and decreasing in the dividend rate, see [22]. These effects have been confirmed in the empirical analysis, see [9, 25, 8], with some exceptions, e.g. see [32].

This way to evaluate a stock option plan suffers of three main drawbacks: a) executives are risk averse and therefore both the value and the effects of this type of compensation computed according to the risk neutral methodology can be misleading, see [14], b) risk neutral evaluation is based on the assumption that assets can be freely traded, this is not the case of executive options, c) the price dynamics is modeled independently of the executive compensation plan, the price of the company follows a dynamics which does not depend on the executive compensation plan, executives cannot interfere with the asset price evolution.

In this paper we address the evaluation of executive stock options by assuming that the executive can manipulate the asset price dynamics. The point we want to make is that executives endowed with an option run the company in order to maximize the expected payoff associated with their option. We simplify the analysis by assuming that executives directly manage the stock price dynamics, and that the asset price is the company value (there is no debt). In this way, we endogenize executives' behavior in the evaluation of their option plan and we capture the asset substitution effect. The market side in determining the asset price evolution is retained in the model by assuming no arbitrage opportunities in the market and evaluating all the compensation plans under the risk neutral martingale measure. To evaluate stock options for executives, we borrow techniques and results from the literature on *passport options*, see [20, 3, 10, 18, 31], and on super-hedging contingent claims in incomplete markets, see [4, 11, 28]. The passport option is an option on a traded account which allows the holder to switch during the life of the option among various positions in the underlying asset, typically the payoff is a classical call: the holder gets the value of his trading account if it is positive and zero if the trading account is negative.

We start by considering an executive endowed with an European call option, when he can manipulate the asset price dynamics by controlling the dividend rate and its volatility in order to maximize the value of the option (expected payoff), an approach similar to the one adopted in some debt pricing papers, see [12, 29]. We show that the optimal strategy coincides with the one obtained according to the Black&Scholes formula: volatility is set to the maximum and the dividend yield to the minimum. These results do not change if executives also own some shares of the company and if they get a number of options increasing in the asset company price (convex payoff).

The second setting is about an executive endowed with a call option on the asset of the company when he is allowed to switch between two policies: a safe policy with the asset price growing at the (constant) risk free rate and a risky project such that the asset price follows a lognormal diffusion process with a drift equal to the risk free rate. In this setting, the executive option is a passport option of the type studied in the above cited papers: the manager gets the value of an imaginary "trading" account (asset price minus the strike) when it is positive and nothing in case of a negative value. By applying results obtained in the passport options literature, we show that the optimal policy is asymmetric. In general, when the option is out of the money, executives adopt the risky policy, when the option is in the money they adopt the risk free policy. The policy depends strongly on the constraints faced by the manager. The above results are confirmed when the executive is endowed with some shares of the company and/or when he gets an amount of options increasing in the company performance. When dividends are not credited to the executive, he has an incentive to set them to the minimum. The case of a project with a negative risk premium is also considered. It is shown that the manager will never adopt it. A bankruptcy condition does not affect considerably the executive's policy; only for a value of the company very close to the bankruptcy level we observe that the agent does not adopt the risky project. When the manager controls only in part the volatility of the asset price, the region characterized by a risky strategy is reduced. The analysis is extended to the case of a manager who can decide between two risky policies and a risk free policy.

We then consider some non-traditional options widely used in executives' compensation, see [22]: premium stock option, performance-vested option, repriceable option, purchased option, reload option, indexed option. We evaluate the executive's policy and the value of this compensation scheme comparing them to those of the classical stock option and to those obtained in [22] with no control by the executives of the asset price. In particular we consider an indexed option. Allowing the executive to manage the volatility of the asset, we observe that the volatility is set to the maximum and that the correlation with the index is set to -1. In general the manager will adopt a risky project when it is negatively correlated with the benchmark and will not when the project is positively correlated.

The analysis described above is characterized by a convex payoff for the executive. Several forms of non-convexities are introduced in executive's compensation plans either to penalize the executive in case of bad performance or to place a cap on his compensation. The behavior of the executive is analyzed by allowing him to switch between the safe policy and the risky policy.

The paper is organized as follows. In Section 2 we address the simplest case of a call option on the asset price with the executive managing the dividend yield and the asset price volatility. In Section 3 we analyze the case of a manager who can decide the percentage of the firm to be run according to a risky policy. In Section 4 we analyze non traditional stock options.

2 European Call Option

We consider an economy with two assets: a risk free asset and the company asset. We assume that the asset can be exchanged in the market and therefore there are no arbitrage opportunities. Under the risk neutral probability measure, the stock price of the company evolves according to the equation

$$dS(t) = S(t)(r - \gamma(t))dt + \sigma(t)S(t)dW(t)), \qquad S(0) = s_0, \tag{1}$$

where $W(t)$ is a standard Brownian motion, r is the risk free rate, $\gamma(t)$ is the dividend rate at time t and $\sigma(t)$ is the volatility of the asset. The manager is endowed with an European call option maturing at time T with a strike price K, the payoff is therefore $[S(T) - K]^+$. The manager can control both the dividend rate $\gamma(t),\ t \in [0, T]$, and the volatility $\sigma(t),\ t \in [0, T]$. We assume that the manager faces the following constraints: $\gamma(t) \in [0, \Gamma]$ and $\sigma(t) \in [\sigma_1, \sigma_2]$ $\forall t \in [0, T]$, $\Gamma > 0$ and $0 < \sigma_1 < \sigma_2$.

The value of this contract at time t is:

$$V(t, S) = \sup_{\gamma(s),\sigma(s)\ t\le s\le T} \left(e^{-r(T-t)} \boldsymbol{E}\left[(S(T) - K)^+ \middle| \mathcal{F}_t \right] \right) \tag{2}$$

subject to $\gamma(s) \in [0, \Gamma]$ and $\sigma(s) \in [\sigma_1, \sigma_2]$. The Hamilton-Jacobi-Bellman equation for $V(t, S)$ is

$$\begin{cases} -rV + V_t + rSV_S + \sup\limits_{\gamma(t)\in[0,\Gamma],\ \sigma(t)\in[\sigma_1,\sigma_2]} \left[-\gamma(t)SV_S + \dfrac{1}{2}\sigma(t)^2 S^2 V_{SS} \right] \\ V(T, S) = (S(T) - K)^+. \end{cases} \tag{3}$$

By employing monotonicity results obtained in [4, 11, 28] on super-hedging a contingent claim with uncertain volatility, or simply by assuming $V_S \geq 0, V_{SS} \geq 0$ and then using the verification theorem, we have that the optimal choice for $\gamma(t), \sigma(t)$ is constant and is given by

$$\begin{cases} \gamma^{opt}(t) = 0 \\ \sigma^{opt}(t) = \sigma_2. \end{cases} \tag{4}$$

Given this result, the price of an executive call option is the Black-Scholes price of a call option with null dividend yield and volatility σ_2.

This simple model confirms what is obtained by applying straight on the Black&Scholes formula to the evaluation of executive options: executives are induced to take more risk, i.e., to increase the volatility of the asset, and to cut dividends. This result has been extended to path dependent contingent claims: the price of a path dependent contingent claim contract whose payoff is increasing in the maximum of the price of the asset in the time interval is increasing in its volatility, see [17].

Note that the above results hold for any contingent claim with a convex payoff. Therefore they are confirmed if executives hold not only a call option but also a positive amount of stocks. For example, let us assume that agents hold $\lambda > 0$ of the stock and $\kappa > 0$ stock options, then the payoff will be $\alpha X(T) + \kappa[X(T) - K]^+$ which is a convex function. The same conclusion can be drawn when the number of options of the stock option plan is an increasing function of the asset price performance. Let α be the number of options if $S(T) \geq K_1 > K$ and β ($\beta < \alpha$) be the number of options if $K \leq S(T) \leq K_1$. Then the payoff for the executive is still convex and therefore the above results hold.

By applying results in [28], the above reasoning can be extended to a call option written on a basket of assets.

3 Risky vs. Safe Projects

Let us consider a manager who can adopt a *safe* policy with the asset price growing at the risk free rate or a *risky* policy with the asset price dynamics described by the lognormal diffusion process (1) with constant coefficients and no dividends. The manager is endowed with a call option written on the asset price expiring at time T. Following this interpretation, the asset price can be interpreted as an account with the owner trading two assets (risk free and risky).

Let $q(t)$ the units of the risky project adopted by the manager at time t, the asset price evolves as follows:

$$dX(t) = q(t)dS(t) + r\left(X(t) - q(t)S(t)\right)dt = rX(t)dt + q(t)\sigma S(t)dW(t), \; X(0) = x_0. \tag{5}$$

To keep the interpretation, we have normalized the company value be equal to its asset price. There is no debt. The executive is endowed with a call option maturing at time T written on X with strike price K. Following [31], we impose the following constraint on the executive's strategy: $q(t) \in [\alpha, \beta]$, $\alpha < \beta$, $\forall t \in [0, T]$. For special cases we have different types of options:

1. Passport call option: $q(t) \in [-1, 1]$;
2. Vacation call option: $q(t) \in [0, 1]$.

In the first case the executive can either dismiss the risky activity of the company or invest in it. In the second case the executive can only invest in it.

The price of the call option at time t is

$$V(t, S, X) = \sup_{q(s)\in[\alpha,\beta],\ t\leq s\leq T} \left(e^{-r(T-t)} \boldsymbol{E}\left[(X(T) - K)^{+} \middle| \mathcal{F}_t \right] \right). \tag{6}$$

Let $Y(t) = X(t) - K$, $Y(t)$ satisfies (5) with initial state $y_0 = x_0 - K$. The price of the contingent claim rewritten in terms of Y, $V(t, S, Y)$ satisfies the Hamilton-Jacobi-Bellman (HJB) equation

$$-rV + V_t + rYV_Y + rSV_S + \frac{1}{2}\sigma^2 \sup_{q(t)\in[\alpha,\beta]} \left(V_{SS} + V_{YY}q^2(t) + 2q(t)V_{SY} \right) \tag{7}$$

In [31] it is shown that the optimal strategy is

$$q^v(t) = \beta \iff Y(t) < \frac{\alpha+\beta}{2}S(t), \quad q^v(t) = \alpha \iff Y(t) > \frac{\alpha+\beta}{2}S(t). \tag{8}$$

Considering a *Passport call Option* we have that the optimal strategy is

$$q^p(t) = 1 \iff Y(t) \leq 0, \quad q^p(t) = -1 \iff Y(t) > 0.$$

When the price is above the strike price of the option, the executive "dismisses" the risky activity in order to invest in the safe activity; if the price is below the strike price of the option, the executive invests in the risky activity.

Considering a *Vacation call Option* we have that the optimal strategy is

$$q^v(t) = 1 \iff Y(t) \leq \frac{1}{2}S(t), \quad q^v(t) = 0 \iff Y(t) > \frac{1}{2}S(t).$$

When the price minus the strike price is below half the value of the risky activity, the executive adopts the risky activity; if the opposite holds, the executive adopts the safe activity.

A contingent claim contract induces a behavior different from that predicted by the Black & Scholes paradigm: an executive will adopt a risky activity only if the asset price is below the strike price (passport option) or if it is smaller than the strike price plus half the value of the risky activity (vacation option). Allowing executives to switch between a safe policy and a risky policy, they behave asymmetrically as far as risk is concerned: when things go well, they reduce risk; when things do not go well, they increase risk. Let $\alpha = 0$, greater is the percentage of the company that can be managed according to the risky policy (β), larger is the region characterized by the risky policy ($Y(t) \leq \frac{\beta}{2}S(t)$). Note that the upperbound of the interval $[0, K + \frac{\beta}{2}S]$ characterized by the risky project as optimal strategy is an increasing function of β and S. This result is due to the fact that a large β and/or S increases the volatility of the company asset when the risky project is chosen: because of the option payoff, a higher volatility of the risky project induces the executive to adopt it for larger values of the asset company. Allowing the executive to manage a large part of the value of the company according to the risky project (high β), he will adopt it until the value of the company has reached a large value.

These results hold true also when the volatility is an increasing function of the asset price, see [18], or stochastic, i.e., the volatility coefficient is driven by a second factor, see [19]. Similar results hold when the constraint is on the value invested in the risky activity.

Let us consider the above problem when the executive can also control the volatility of the risky project. Let us consider equation (1) with $\gamma(t) = 0$, $\sigma(t) \in [\sigma_1, \sigma_2]$, $t \in [0, T]$. The value of the option is

$$V(t, S, X) = \sup_{\substack{q(s)\in[\alpha,\beta] \\ \sigma(s)\in[\sigma_1,\sigma_2]}} {}_{t\leq s\leq T} \left(e^{-r(T-t)} \boldsymbol{E}\left[(X(T) - K)^+ \middle| \mathcal{F}_t \right] \right). \tag{9}$$

We easily obtain that the strategy for q does not change and volatility is set to the maximum:

$$\begin{cases} \sigma^{opt}(t) = \sigma_2 \\ q^{opt}(t) = \alpha I_{\{Y(t) \geq \frac{\alpha+\beta}{2} S(t)\}} + \beta I_{\{Y(t) < \frac{\alpha+\beta}{2} S(t)\}}. \end{cases} \tag{10}$$

To show this fact, we use the verification Theorem and a result obtained in [17, Lemma 4.3] establishing that $V_{SS} + 2q^{opt}(t)V_{SY} + (q^{opt}(t))^2 V_{YY} \geq 0$, where $q^{opt}(t)$ is given by (8) and V is the associated solution. Then it is easy to verify that the candidate solution satisfies the HJB equation associated with problem (9):

$$rV - V_t - rSV_S - rYV_Y = \sup_{q(t)\in[\alpha,\beta], \sigma(t)\in[\sigma_1,\sigma_2]} \frac{1}{2}\sigma^2(t) s^2 (V_{SS} + 2q(t)V_{SY} + q^2(t)V_{YY}). \tag{11}$$

Let us now consider the case of an executive endowed with $\kappa > 0$ European options and $\lambda > 0$ stocks of the company. The executive's payoff is $\lambda X(T) + \kappa[X(T) - K]^+$ or, equivalently, $\lambda Y(T) + \kappa Y(T)^+ + \lambda K$. The value of this payoff at time t is:

$$V(t, S, X) = \lambda(y_0 + Ke^{-r(T-t)}) + \kappa e^{-r(T-t)} \sup_{\substack{q(s)\in[\alpha,\beta] \\ \sigma(s)\in[\sigma_1,\sigma_2]}} {}_{t\leq s\leq T} \boldsymbol{E}\left[Y(T)^+ \right]. \tag{12}$$

The problem can be reduced to the previous one obtaining the optimal strategy (10). The above result also holds true when the number of options increases with the performance of the company. Let α be the number of options if $S(T) \geq K_1 > K$ and β $(0 < \beta < \alpha)$ be the number of options if $K \leq S(T) \leq K_1$. Then the payoff is still convex and therefore we can apply the results in [15, 31] to show that the optimal policy is the one described above. The same conclusion can be drawn if the manager is compensated through a fixed wage plus a stock option.

When the executive can also manage the instantaneous dividend rate, equation (5) becomes

$$dX(t) = (r - \gamma(t))\, X(t)dt + q(t)\sigma S(t)dW(t), \tag{13}$$

where $\gamma(t)$ is the dividend rate at time t. When dividends are not credited to executives and they manage the dividend rate under the constraint $\gamma(t) \in [0, \Gamma]$, $\forall t \in [0, T]$, the optimal policy is the one obtained above plus $\gamma(t) = 0$, $\forall t \in [0, T]$. This fact is easily shown by means of the verification theorem for the HJB equation (11) with $(r - \gamma(t))YV_Y$ instead of rYV_Y. This result does not hold if the executive is credited for the dividends.

The welfare effect of stock option plans is strongly disputed. In particular, an important point is the following: with a stock option, does the executive undertake projects with negative premium, i.e., a drift lower than the risk free rate?

To answer this question, we consider the following dynamics for the value of the risky project:

$$dS(t) = S(t)(r + p)dt + \sigma S(t)dW(t)), \tag{14}$$

where p is a constant representing the premium of the project. Note that an executive aiming to maximize the expected value of the asset at time T will never chose the risky project with $p < 0$.

By considering a company whose asset is exchanged in the market we require $e^{-rt}X(t)$ to be a martingale under the risk neutral probability measure. Therefore, changing the measure through the Girsanov theorem, we are led to a new Brownian motion such that the drift for $X(t)$ is r and all the analysis developed above holds. As the option is evaluated under the risk neutral probability measure, the risk premium of the risky project does not affect agent's decisions: the manager will adopt the risky project also with a negative risk premium when the above conditions apply.

Let us consider now a firm whose asset is not exchanged in the market. In this case no change of measure is needed and as a consequence, assuming a risk neutral executive the HJB equation associated with the executive problem (6) becomes

$$\begin{cases} -rV + V_t + (r+p)SV_S + rXV_X + \sup\limits_{q(t)\in[\alpha,\beta]} \left[pq(t)SV_Y + \frac{1}{2}\sigma^2 \left(V_{SS} + V_{YY}q^2(t) + 2q(t)V_{SY}\right)\right] \\ V(T, S, Y) = Y^+. \end{cases} \tag{15}$$

The analysis of this problem cannot be developed in closed form, we reply upon a numerical analysis discretizing (15). When $p < 0$, the executive will never adopt the risky project. Therefore, executives' options do not have a negative welfare effects as far as investment decisions are concerned.

Allowing the executive to control the diffusion process of the asset price, the value of an European call option is different from the value obtained through the Black and Scholes formula. In Fig. 1 we compare the value of the vacation call option to the value of an European call option. It is shown that the value of the vacation call is higher than the Black and Scholes value, the difference is increasing in the volatility and is particularly large for in the money options.

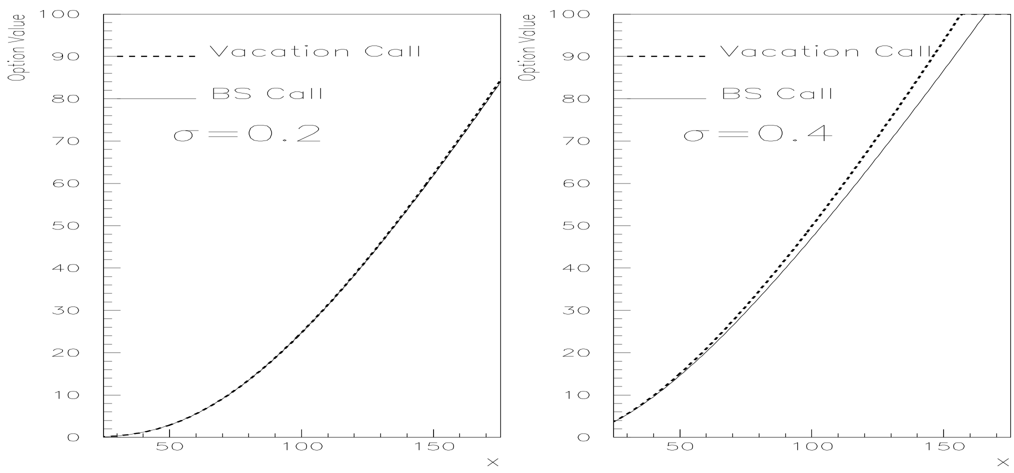

Figure 1: Comparison between a Call Option and a Vacation Call with $K = 100$, $\sigma = 0.2$ (left) and $\sigma = 0.4$ (right), as a function of $X(0) = S(0)$.

Introducing a bankruptcy condition, the analysis does not change too much. Let us assume that when $X = 0$ is reached the company is closed and the manager endowed with a call option on the company asset gets -100 (i.e., loss of the fixed wage). The problem faced by the manager is the one in (6) with the bankruptcy condition, the HJB equation is given in (7) with the boundary condition $V(t, S, 0) = -100$. The option is a *knocked out passport barrier option*. Using a numerical approach, we evaluate the manager strategy. Results are illustrated in Figure 2: only for very low value of X the manager will not adopt the risky investment, otherwise the policy is that depicted in case of the standard option.

In the above setting we assumed that the manager completely manage the volatility of the asset price. In what follows we assume the presence of idiosyncratic noise in the asset price evolution, as a consequence the asset price evolves as follows:

$$dX(t) = rXdt + qS\sigma dW(t) + X\sigma_1 dW_1(t) \tag{16}$$

where dW_1 is uncorrelated with dW and σ_1 is a positive constant. The optimal strategy with a bankruptcy condition is shown in Figure 3.

The region characterized by a risky policy is smaller than the one associated with a vacation call option. In particular, the manager will not adopt the risky strategy under financial distress conditions and for a high asset price (in the money option).

We assume now that the executive can adopt two different risky activities, the value of these activities evolves as follows:

$$\begin{aligned} dS_1(t) &= rS_1(t)dt + \sigma_1 S_1(t)dW_1(t) \\ dS_2(t) &= rS_2(t)dt + \sigma_2 S_2(t)dW_2(t), \end{aligned} \tag{17}$$

where W_1 and W_2 are two Brownian motions, for simplicity we assume them not to be correlated, and $0 < \sigma_1 < \sigma_2$, i.e., the second project is riskier than the first one.

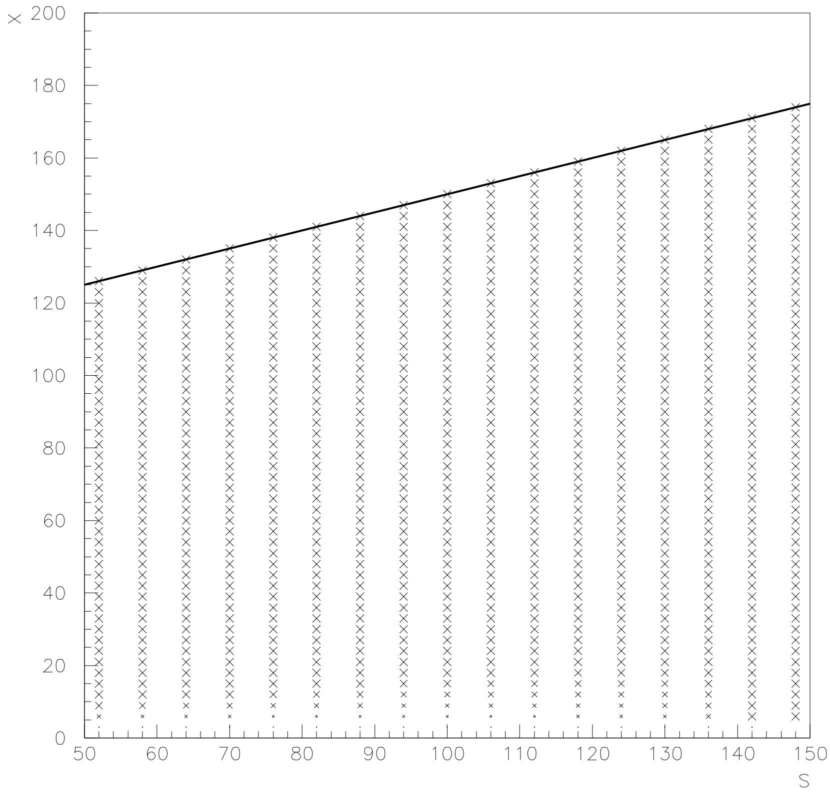

Figure 2: Points (X, S) with $q > 0$ and a bankruptcy condition such that V(t,S,0) = -100, at $t = 5$ years, when the executive holds a traditional stock option with strike price 100. Here and in the following figures the magnitude of the cross is proportional to q. The solid line represents the theoretical boundary between the regions in which $q = 1$ and $q = 0$ for the traditional stock option (without bankruptcy).

The manager can choose to invest $q_1 \in [0, 1]$ in asset 1 and $q_2 \in [0, 1]$ in asset 2, under the constraint $q_1 + q_2 \leq 1$. The stock price will evolve as follows:

$$dX = rX\,dt + q_1(t)\sigma_1 S_1 dW_1 + q_2(t)\sigma_2 S_2 dW2. \tag{18}$$

The manager aims to maximize the option value. Following [2], we can reduce the dimensionality of the problem by setting:

$$Z_1 = \frac{X}{S_1}, \qquad Z_2 = \frac{S_2}{S_1}.$$

We also define a new probability measure $\tilde{Q}$ such that:

$$\frac{d\tilde{Q}}{dQ} = exp(\sigma_1 W_1(T) - \frac{1}{2}\sigma_1^2 T), \qquad \tilde{W}_1(t) = W_1(t) - \sigma_1 t.$$

Then by straightforward computation we have

$$\begin{aligned} dZ_1 &= \sigma_1(q_1 - Z_1)d\tilde{W}_1 + q_2\sigma_2 Z_2 dW_2 \\ dZ_2 &= Z_2(-\sigma_1 d\tilde{W}_1 + \sigma_2 dW_2) \end{aligned} \tag{19}$$

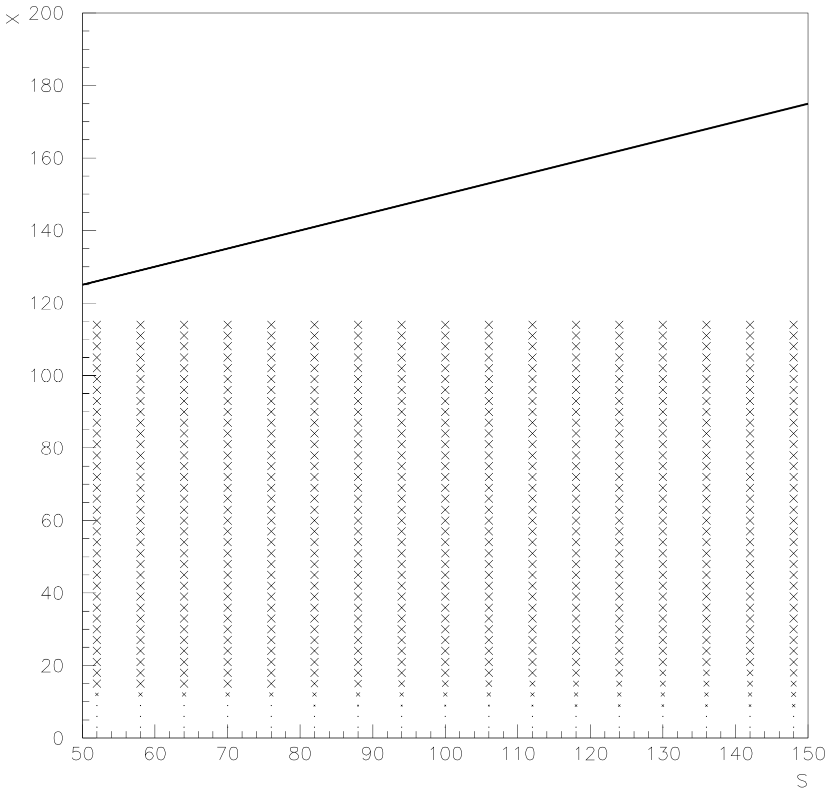

Figure 3: Points (X, S) with $q > 0$ when $\sigma = \sigma_1 = 0.2$ at $t = 5$ years when the executive a stock option with strike 100 and the asset price follows (16).

and the option value becomes

$$V(Z_1, Z_2, t) = S_1(t) \cdot \sup_{q_1, q_2} \tilde{\boldsymbol{E}}\left[Z_1^+(T)\right]. \tag{20}$$

The corresponding HJB equation is

$$\begin{cases} V_t + \frac{1}{2}V_{Z_2Z_2}Z_2^2(\sigma_1^2 + \sigma_2^2) + V_{Z_1Z_2}\sigma_1^2 Z_1 Z_2 + \\ + \sup_{q_1,q_2}\left[\frac{1}{2}V_{Z_1Z_1}\left(\sigma_1^2(q_1 - Z_1)^2 + q_2^2\sigma_2^2 Z_2^2\right) + V_{Z_1Z_2}\left(-\sigma_1^2 Z_2 q_1 + q_2\sigma_2^2 Z_2^2\right)\right]; \\ V(Z_1, Z_2, T) = Z_1^+. \end{cases} \tag{21}$$

We solve this equation numerically; the resulting strategy is shown in Figure 4 and 5. The executive always adopts a risky strategy. The risk-free policy is substituted by the project with smaller volatility $(\sigma_i S_i)$. The riskiest project is adopted when the asset price is low. The region characterized by the riskiest strategy is smaller than that obtained with a risk-free project.

4 Non-traditional options

In this section we analyze non-traditional stock options considered in [22]. We will consider a risky project starting from $S(0) = 100$ with a volatility $\sigma = 0.2$ and $r = 0$. The strike price

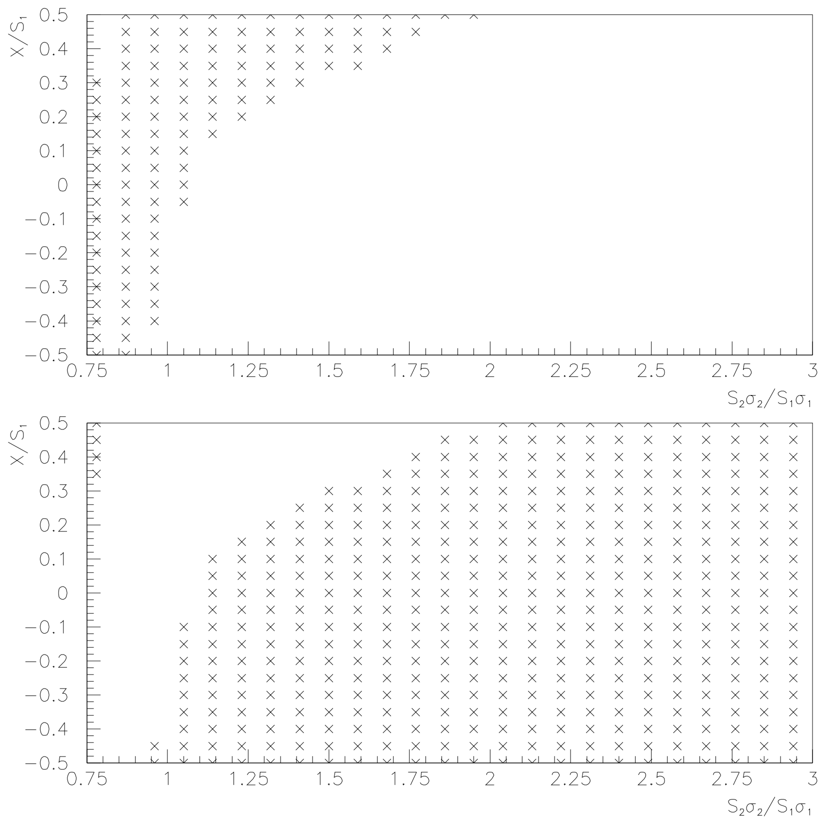

Figure 4: Points (X, S) with $q_1 = 1$ (top) and $q_2 = 1$ (bottom) when $\sigma_1 = 0.1$ and $\sigma_2 = 0.15$ at $t = 5$ years when the executive a stock option with strike 100.

will be set to $K = 100$. The maturity of the granted stock option is ten years.

4.1 Premium stock option

The premium stock option is a traditional stock option with a strike price higher than the granting day asset price, which we set to $K' = 150$. Without bankruptcy, the executive's strategy and the option value can be obtained in a closed form according to [31]. This type of compensation plan induces the agent to take more risk: he adopts the risky strategy in an (S,X) region larger than in the case of an option with strike price equal to the granting day price.

4.2 Performance-vested stock option

The performance vested option is an up and in barrier option; it is a traditional stock option which becomes exercisable only if the stock price hits a prescribed level $\overline{X}$. The equation to be solved numerically is (7) for $V(t, S, X)$ defined in the region $[0, T] \times [0, +\infty] \times [0, \overline{X}]$ with the boundary condition:

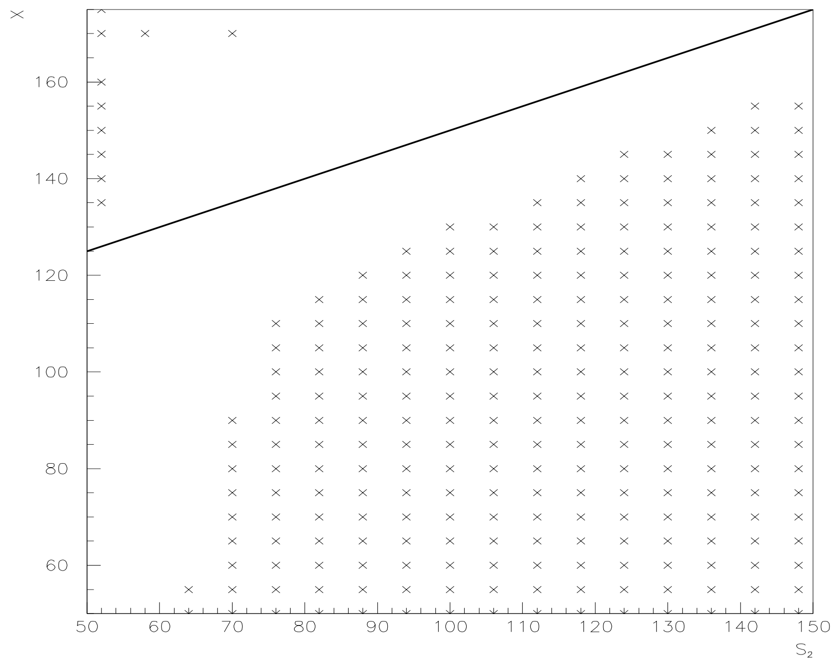

Figure 5: Points (X, S) with $q_2 = 1$ (the riskiest activity) when $\sigma_1 = 0.1$ and $\sigma_2 = 0.15$ at $t = 5$ years with $S_1(t) = 100$ when the executive holds a stock option with strike 100 and can switch between two assets.

$$\begin{cases} V(T, S, X) = 0 \\ V(t, S, \overline{X}) = VC(t, T, \sigma, K, S, \overline{X}) \end{cases} \tag{22}$$

where $VC(t, T, \sigma, K, s_0, x_0)$ denotes the value defined in [31] of a Vacation Call at time t with maturity at $T - t$, volatility σ, strike K and starting values $S(t) = s_0$, $X(t) = x_0$. The resulting strategy with $\overline{X} = 150$ is to set $q = 1$ until the barrier is reached; then the policy of the traditional vacation option applies. As a consequence, a vested option induces the executive to always adopt a risky strategy before reaching the barrier.

4.3 Repriceable stock option

Repriceable stock options have been proposed to retain talented executives after a stock price decline. It is like a traditional stock option, but if the price goes under a given level (X_{low}) then the stock price becomes $K' < K$. The equation to be solved is again (7) in the region $[0, T] \times [0, +\infty) \times [X_{low}, +\infty)$ with the boundary conditions:

$$\begin{cases} V(T, S, X) = (X - K)^+ \\ V(t, S, X_{low}) = VC(t, T, \sigma, K', S, X_{low}). \end{cases} \tag{23}$$

The resulting strategy with $X_{low} = 50$, $K' = 50$ is shown in Figure 6.

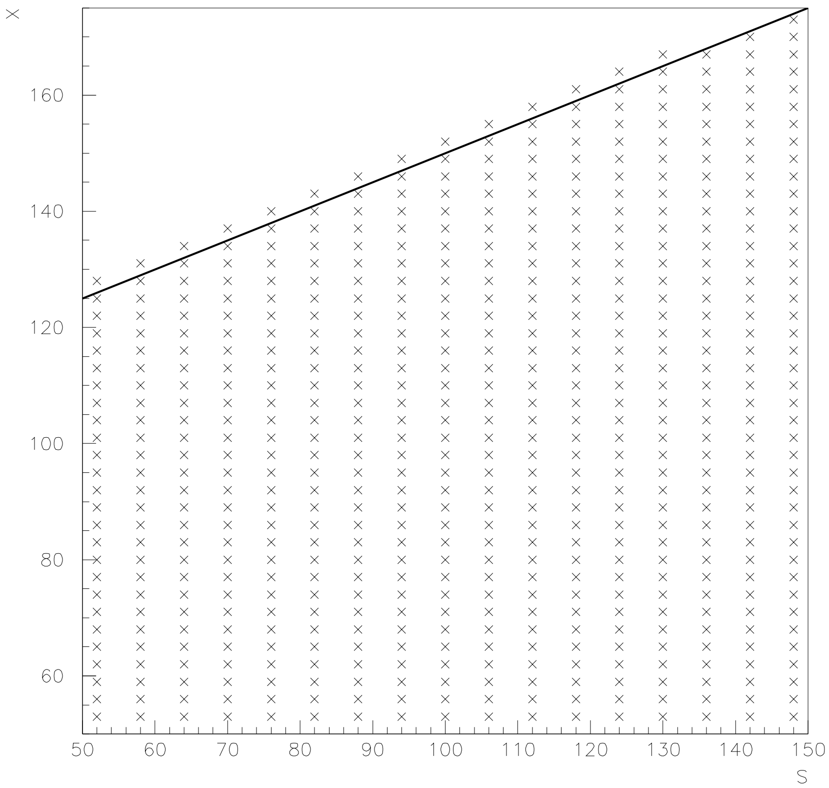

Figure 6: Points (X, S) with $q > 0$ at $t = 5$ years when the executive holds a repriceable stock option with the strike resetted to 50 if the stock price falls to 50. The solid line represents the theoretical boundary between the the regions in which $q = 1$ and $q = 0$ for the traditional stock option.

The Figure shows that the manger is induced to adopt a policy riskier than the one associated with the standard option. The rationale for this result is very simple: as there is an opportunity of repricing the option the manager does not care of a bad performance.

4.4 Purchased stock option

In the purchased stock option, the executive has to prepay a fraction α of the strike price. If the option expires out of the money, the executive looses that amount of money. The evaluation of this stock option plan can be addressed trough equation (7) with the final payoff

$$V(T, S, X) = \begin{cases} X - K & for\ X \geq K \\ -\alpha K & for\ X < K. \end{cases} \tag{24}$$

The resulting strategy with $\alpha = 0.1$ is shown in Figure 7.

The executive will play a policy less aggressive than the one associated with a traditional vacation call. The region characterized by the risky strategy is smaller than the one associated with that contract.

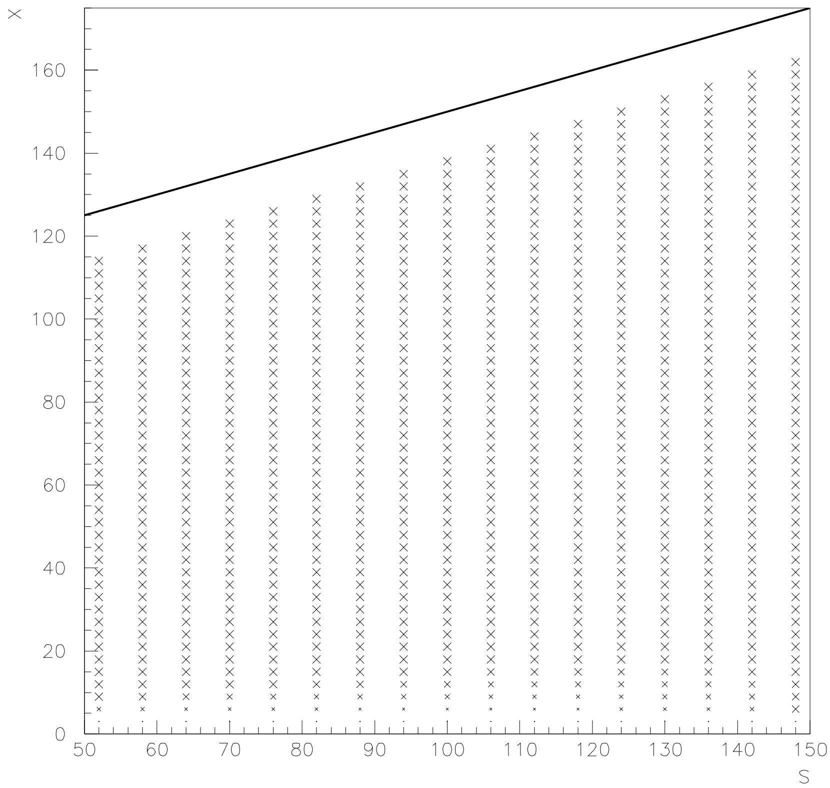

Figure 7: Points (X, S) with $q > 0$ at $t = 5$ years when the executive holds a purchased stock option with a prepay of 10% of the strike price. The solid line represents the theoretical boundary between the the regions in which $q = 1$ and $q = 0$ for the traditional stock option.

4.5 Reload stock option

A reload stock option establishes that the manager pays the strike price of the exercised option by tendering shares he already owns, which are valued at their market price when exercised, in exchange the manager receives new options (reload). We make the simplifying hyphotesys that the manager can exercise the option only at $T_1 < T$. The equation to be solved is (7) in the region $[0, T_1] \times [0, +\infty], [0, +\infty]$ with the boundary condition:

$$V(T_1, S, X) = \begin{cases} VC(T_1, T, \sigma, K, S, X) & for\ X < K \\ X - K + \frac{K}{X} VC(T_1, T, \sigma, X, S, X) & for\ X \geq K \end{cases} \tag{25}$$

The resulting strategy with $T_1 = 5$ years is shown in Figure 8.

Again the manager is induced to play a less aggressive strategy with respect to what is observed with a vacation call option.

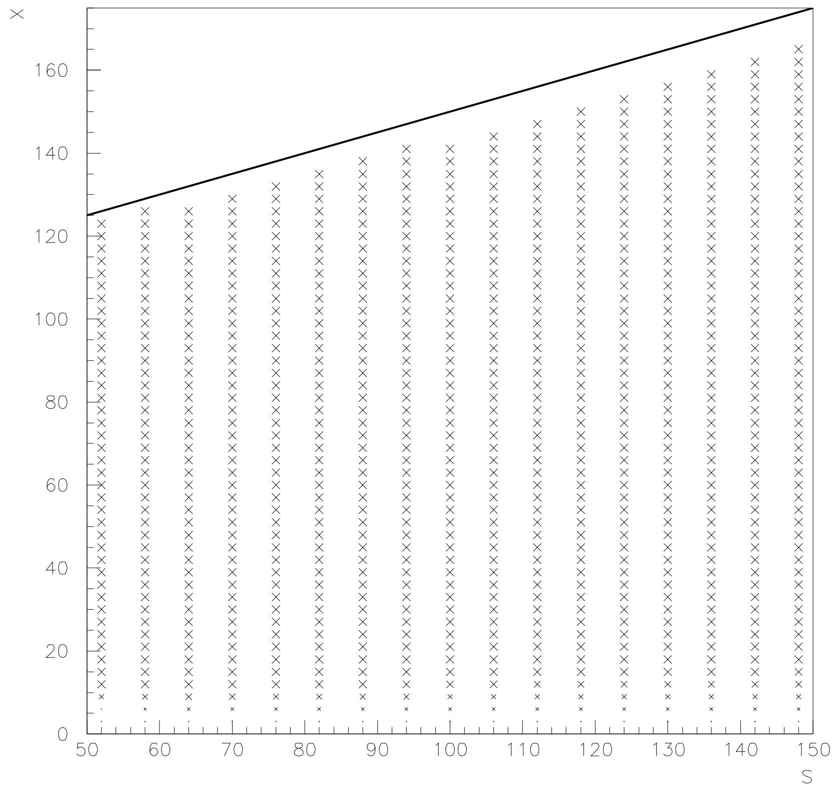

Figure 8: Points (X, S) with $q > 0$ at $t = 2.5$ years when the executive holds a reload stock option, with the grant to "reload" the option at $t = 5$ years. The solid line represents the theoretical boundary between the the regions in which $q = 1$ and $q = 0$ for the traditional stock option.

4.6 Indexed Option

In the setting described in Section 2, we consider an indexed option of the type analyzed in [23]. The payoff of the contract at time T is

$$[S(T) - \lambda Z(T)]^+, \ \lambda > 0,$$

where $S(t)$ is the company asset price and $Z(t)$ is the benchmark of the manager's compensation (index), the two state variables evolve according to the following system of stochastic differential equations:

$$\begin{aligned} dS(t) &= rS(t)dt + S(t)(\sigma_{11}(t)dW_1(t) + \sigma_{12}(t)dW_2(t)) \\ dZ(t) &= rZ(t)dt + Z(t)(\sigma_{21}(t)dW_1(t) + \sigma_{22}(t)dW_2(t)). \end{aligned} \tag{26}$$

The executive is endowed with an indexed option and can manage the volatility of the stock price $(\sigma_{11}(t),\ \sigma_{12}(t))$, under some constraints. The executive maximizes the value of the contingent claim contract. In [28] it is shown that the executive attains his goal by setting $\sigma_{11}(t)+\sigma_{12}(t)$ to the maximum under the constraint. Therefore the results obtained in Section 2 for a classical call option are confirmed. In the above paper it is shown that when the executive can manage the correlation between the index and the stock of the company, the maximum of the value of the contract is attained by setting the correlation between $S(t)$ and $Z(t)$ constant and equal to -1. When the executive is remunerated by the performance of the asset price with respect to a benchmark, he has an incentive to have an asset negatively correlated with the benchmark. Moreover, it is easy to verify through the verification theorem for the HJB equation associated to the problem that the dividend rate should be set equal to the minimum allowed by the constraints.

Let us consider now the case of an executive who can manage the value of the company $X(t)$ by adopting a risk free policy or a risky policy such that the value of the company follows the index evolution. The value of the company evolves as

$$dX(t) = q(t)dZ(t) + r\,(X(t) - q(t)Z(t))\,dt = rX(t)dt + q(t)\sigma Z(t)dW(t),\ \ X(0) = x_0 \qquad (27)$$

where

$$dZ(t) = rZ(t)dt + \sigma Z(t)dW(t)$$

and the payoff is

$$[X(T) - Z(T)]^+.$$

As above we assume that $q(t) \in [\alpha, \beta]$. Set $Y(t) = X(t) - Z(t)$, it is easy to show that the optimal management of the company can be reduced to that of the passport option yielding the optimal policy described in (8). As a consequence, when $\alpha = 0$ we have

$$q^{opt} = 0 \iff X(t) - Z(t) > \frac{\beta}{2}Z(t),\ \ q^{opt} = \beta \iff X(t) - Z(t) \leq \frac{\beta}{2}Z(t).$$

If the asset of the company is well above the benchmark, the executive manages the company adopting the risk free policy, otherwise he follows the index amplifying its dynamics. The interval $[0, Z(1+\beta/2)]$ describing the region for which it is optimal to follow the index is increasing in β and Z. As in the non indexed option, this phenomenon is due to the fact that the option payoff is asymmetric. As in that case, below the benchmark it is always optimal to adopt the risky policy.

Let us consider now the case of an executive who can adopt a risky activity ($S(t)$) which is not perfectly correlated to the benchmark. We assume that the benchmark evolves as:

$$dZ(t) = rZ(t)dt + \overline{\sigma} Z(t)dW_1(t).$$

and the risky activity evolves as

$$dS(t) = rS(t)dt + \rho\sigma S(t)dW_1 + \sqrt{1-\rho^2}\sigma S(t)dW_2(t), \qquad (28)$$

where W_1, W_2 are two independent Brownian motions. The company price evolves as follows:

$$\begin{aligned} &dX(t) = q(t)dS(t) + r\,(X(t) - q(t)S(t))\,dt = \\ &rX(t)dt + q(t)[\rho\sigma S(t)dW_1(t) + \sqrt{1-\rho^2}\sigma S(t)dW_2(t)],\ \ X(0) = x_0, \end{aligned} \qquad (29)$$

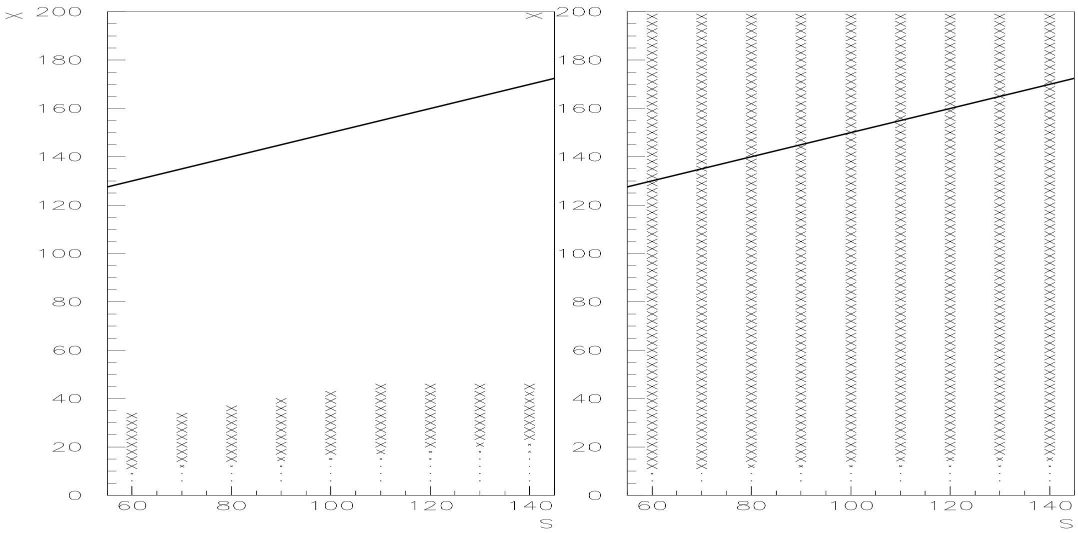

Figure 9: Points (X, S) with $q > 0$ at $t = 5$ years and $Z(t) = 100$ when the executive holds an indexed stock option with $\rho = 0.75$ (left) and $\rho = -0.25$ (right). The solid line represents the theoretical boundary between the the regions in which $q = 1$ and $q = 0$ for the traditional stock option.

the constraint on the strategy is $q(t) \in [\alpha, \beta]$, the executive's payoff is as above and the HJB equation reads:

$$-rV + V_t + rZV_Z + rSV_S + rXV_X + \tfrac{1}{2}Z^2\overline{\sigma}^2V_{ZZ} + \tfrac{1}{2}\sigma^2S^2V_{SS} + \rho\sigma\overline{\sigma}SZV_{SZ}+ \\ + \sup_{q(t)\in[\alpha,\beta]} \left(\frac{1}{2}q^2\sigma^2S^2V_{XX} + q\sigma^2S^2V_{SX} + q\rho\sigma\overline{\sigma}SZV_{ZX}\right) = 0. \tag{30}$$

Figure 9 shows the strategy of the executive when the index is positively and negatively correlated with the project. When the correlation is negative the manager adopts the risky activity unless the asset price is near to the bankruptcy level. When the correlation is positive the manager will adopt the risky policy only for very low level of the asset price.

This result confirms the above findings.

4.7 Call-Spread stock option

The Call-Spread stock option has the following final payoff:

$$\begin{cases} [X(T) - K]^+, & X(T) \le \overline{X} \\ \overline{X} - K, & X(T) > \overline{X}, \end{cases} \tag{31}$$

where X is the asset price and $0 < K < \overline{X}$. This type of payoff is built to provide an incentive for the executive up to $\overline{X}$. Being the payoff no convex, results in [31] cannot be applied. We tackle this problem by formulating an implicit solution, then evaluating the solution numerically. The resulting strategy with $\overline{X} = 150$ is shown in Figure 10. Analyzing the region characterized by

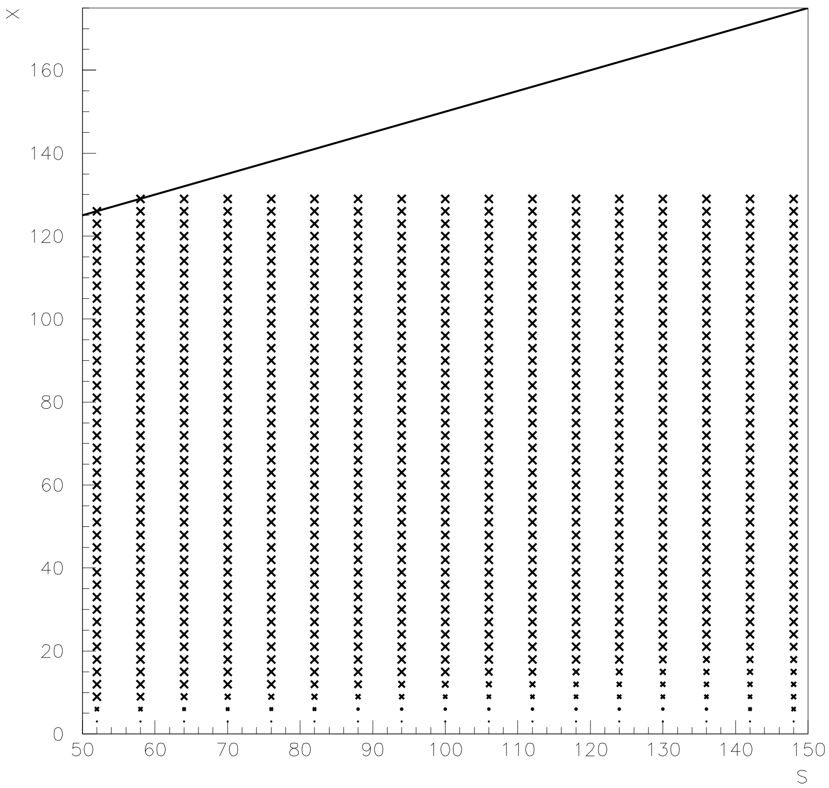

Figure 10: Points (X, S) with $q > 0$ at $t = 5$ years when the executive holds a Call-Spread stock option with final payoff (31). The solid line represents the theoretical boundary between the the regions in which $q = 1$ and $q = 0$ for the traditional stock option.

adoption of the risky project, we have that it is contained in the one associated with a classical option. Obviously when $X \geq \overline{X}$ the agent does not take any risk.

4.8 Penalty stock option

We can introduce non-convexity in the stock option plan by penalizing the executive in case of bad performance and cutting the fixed wage when the performance is not satisfactory. The following payoff is typically obtained:

$$\begin{cases} [X(T) - K]^+, & X(T) \geq K_1 \\ B, & X(T) < K_1, \end{cases} \tag{32}$$

where B is a negative constant. As for the previous payoff, we evaluated the solution numerically with $B = -10, K = 100, K_1 = 70$. The resulting strategy is plotted in Figure 11. As for the bankruptcy condition, the executive strategy is sensitive to the penalty only next to 70.

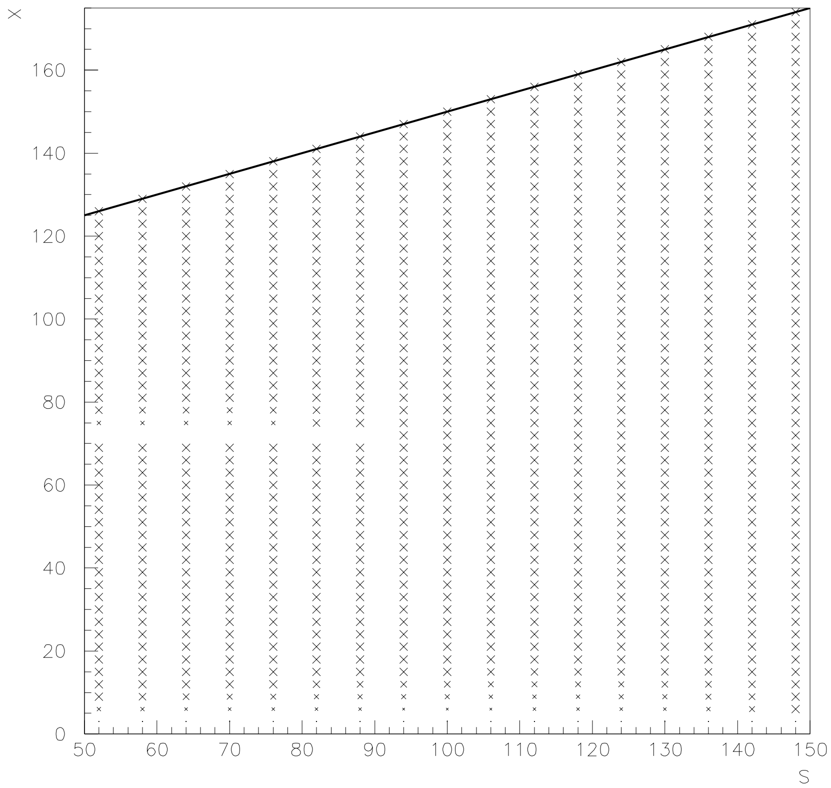

Figure 11: Points (X, S) with $q > 0$ at $t = 5$ years when the executive holds a penalty stock option. The solid line represents the theoretical boundary between the the regions in which $q = 1$ and $q = 0$ for the traditional stock option.

5 A Comparison and Conclusions

We analyze the option payoffs on three different points:

- investment strategy;
- vacation call option value-non traditional option value
- non traditional option value according to Black&Scholes techniques and with control of the diffusion process.

On the first point we have that a call option induces the agent to adopt a risky strategy. Differently from what is obtained through classical comparative statistic exercises, with the classical Black & Scholes formula, the optimal policy is asymmetric: the executive adopts a risky project when the option is out of the money and in the money but next to the strike price. The agent is weakly sensitive to the bankruptcy condition but he does not adopt a project with a negative risk premium. A repriceable stock option, a vested stock option and a premium stock option induce the agent to take more risk than in a vacation call option, a purchased and a reload stock option induce the executive to take less risk. The behavior of a manager endowed with an index option is strictly dependent on the degree of correlation between the project and the benchmark: the executive will adopt a risky strategy when the project is negatively correlated with the benchmark.

These results are only in part similar to what is obtained in [22] on the sensitivity of the value of the option to a change in the volatility. The main differences are that an index option with a positive correlation induces the manager to take less risk than a traditional option and the same result holds for a reload option.

As far as the price of non traditional options compared to the vacation call value is concerned, we have that a traditional option gives a value higher than a premium, vested, purchased, call-spread and penalty option. A repriceable option instead gives a value higher than a traditional vacation option. An indexed option with a negative correlation has less value than a traditional option, an indexed option with a positive correlation instead has more value. See Figure 12. These results confirm those obtained in [22] with classical no arbitrage techniques.

As for a classical European option, allowing the executive to control the asset price dynamics, the value of the options payoffs increase with respect to what is obtained according to classical techniques. This phenomenon is particularly acute for a vested option and an indexed option, see Figure 13.

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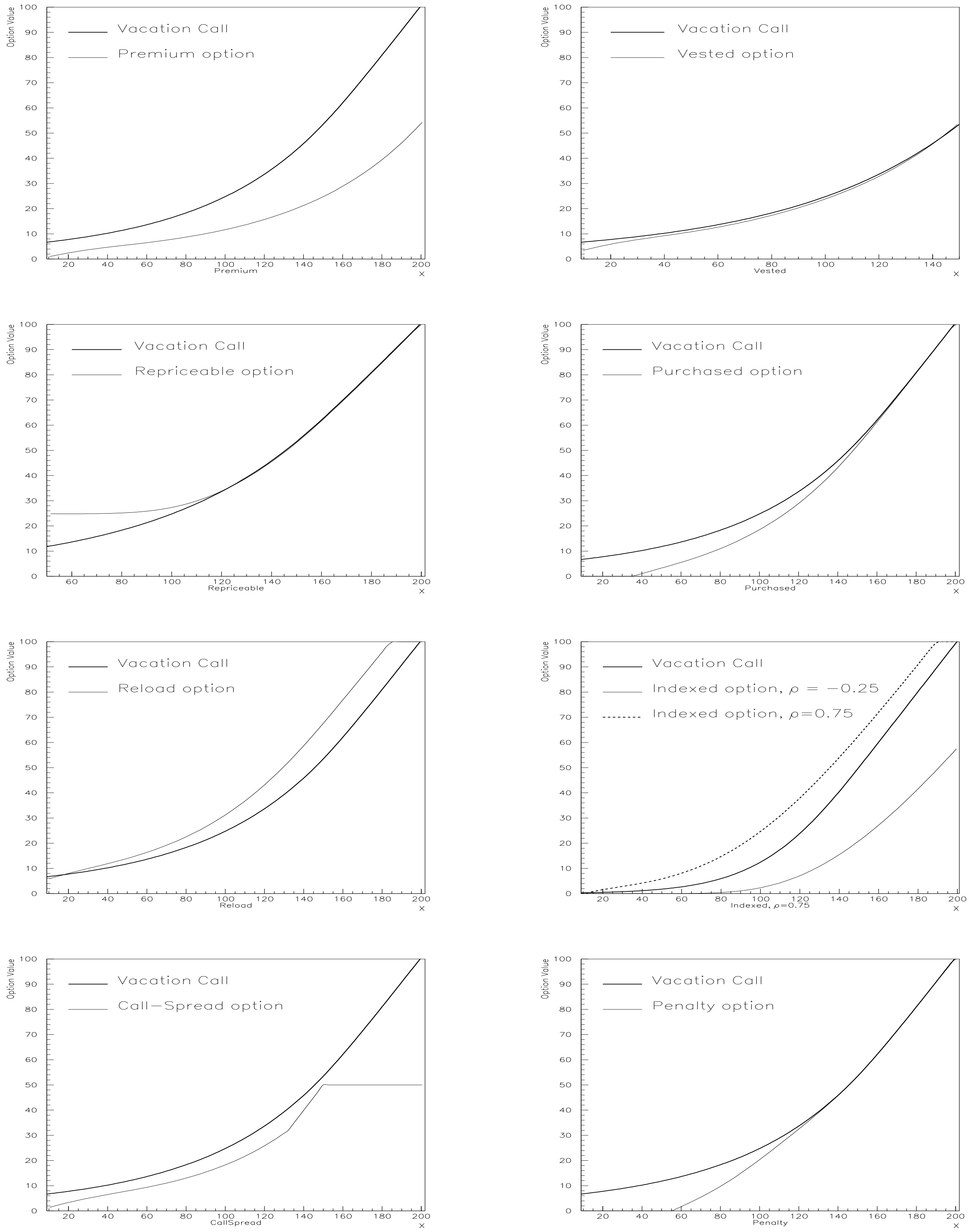

Figure 12: Comparison of the value of non traditional options and of the value of a classical European call option allowing the executive to control the diffusion process.

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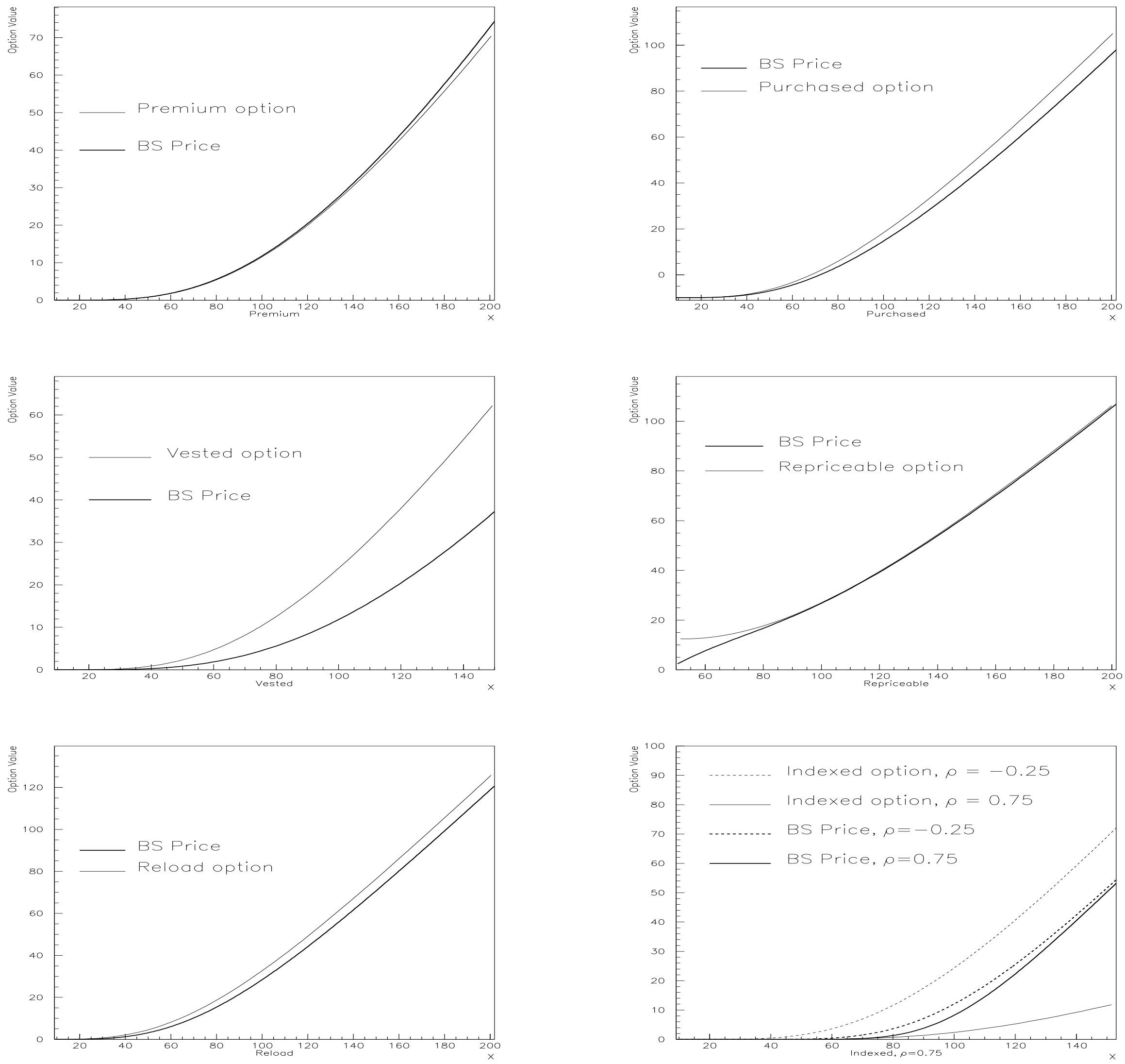

Figure 13: Comparison of the value of non traditional options allowing the executive to control the volatility and according to classical no arbitrage techniques (BS price), as a function of $X(0) = S(0)$.

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