Unexpected volatility and intraday serial correlation*

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Abstract

We study the impact of volatility on intraday serial correlation, at time scales of less than 20 minutes, exploiting a data set with all transactions on SPX500 futures from 1993 to 2001. We show that, while realized volatility and intraday serial correlation are linked, this relation is driven by unexpected volatility only, that is by the fraction of volatility which cannot be forecasted by a linear model. The impact of predictable volatility is instead found to be negative (LeBaron effect). Our results are robust to microstructure noise, and they confirm the leading economic theories on price formation.

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1 Introduction

The study of serial correlation in asset prices is of great importance in financial economics. Indeed, from the point of view of market efficiency (Fama, 1970), as well as market inefficiency (Shleifer, 2003), serial correlation is a market anomaly which needs to be addressed by economic theories. Once serial correlation is significantly detected in the data, see James (2003) as an example, an explanation is needed to reconcile the empirical finding with the assumption of informational efficiency of the market. This has been typically accomplished in a rational setting (Lo and MacKinlay, 1990; Boudoukh et al., 1994; Sentana and Wadhwani, 1992; Safvenvblad, 2000) or in a behavioral setting (Cutler et al., 1991; Jegadeesh and Titman, 1993; Chan, 1993; Badrinath et al., 1995; Challet and Galla, 2005). In this paper, we concentrate on very short-run serial correlation, that is we focus on intraday data and in particular on time scales from 4 to 20 minutes.

The purpose of this paper is multiple. Beyond showing the informational efficiency of the considered market, which is actually out of discussion given its liquidity, our aim is to study the dynamical properties of intraday serial correlation. We extend previous literature by decomposing intraday volatility, measured by means of realized volatility, into its predictable and unpredictable part. To quantify intraday serial correlation, we use the variance-ratio test on evenly sampled intraday data. While being very standard for daily data, the variance ratio test has still little application on high-frequency data, including Andersen et al. (2001); Thomas and Patnaik (2003); Kaul and Sapp (2005).

Our main result is that intraday serial correlation is positively linked with *unexpected volatility*, defined as the residual in a linear regression model for daily volatility as measured with intraday data. In other words, unexpected volatility is that part of volatility which was not forecasted (using a linear model) on that market in that particular day. We also explain the puzzling results of Bianco and Renò (2006) who, on a much less liquid market (Italian stock index futures), found volatility to be positively correlated with serial correlation, at odds with the results in LeBaron (1992). We show that indeed total volatility is positively related to serial correlation: however, it is unexpected volatility that drives this positive relation. The predictable part of volatility, used in LeBaron (1992) via a GARCH-like filter, turns out to be negatively related to serial correlation, in agreement with previous literature.

The paper is organized as follows. Section 2 illustrates the methodology and describes the data set. Section 3 shows the estimation results and discusses the implications of them. Section 4 analizes the robustness of the statistical analysis against specific exceptions highlighted by the characteristics of the data and the analysis itself. Section 5 concludes.

2 Data and methodology

The data set under study is the collection of all transactions on the S&P500 stock index futures from April, 1993 to October 2001, for a total of 1,975 trading days. We have information on all futures maturity, but we use only next-to-expiration contracts, with the S&P 500 expiring quarterly. We use only transactions from $8:30$ a.m. to $3:15$ p.m.. In total, we have $4,898,381$ transactions, that is $2,480$ per day on average, with an average duration between adjacent trades of $9,8$ seconds. Not all high-frequency information is used. We use instead a grid of evenly sampled data every day. We find that a time interval of $\Delta t = 4$ minutes is a large enough to avoid the problem of intervals with no price changes within. Thus, for every day, we have a time series of 101 evenly sampled prices. Then, we do not use high frequency information directly, but we aggregate the intraday time series to daily time series of the quantities of interest. Basically, we want to estimate serial correlation and volatility, and we use a transformation of the variance ratio for the first, and realized volatility for the second.

To study intraday serial correlation, we use a correction of the variance-ratio statistics. We implement the variance ratio test according to the heteroskedastic consistent estimator (Lo and MacKinlay, 1988) with overlapping observations (Richardson and Smith, 1993), for which the asymptotic distribution is well known under the null. This briefly consists in what follows. Denote by $P_k,\ k = 1, \ldots, N$ a time series (log-prices) and define the first differences time series $r_k = P_k - P_{k-1}$ (log-returns). Suppose to have a set of $N = nq + 1$ observations, where q is an integer greater than 1. We then define:

$$\hat{\mu} \equiv \frac{1}{nq}\sum_{k=1}^{nq}(P_k - P_{k-1}) = \frac{1}{nq}(P_{nq} - P_0) \tag{1}$$

$$\hat{\sigma}_a^2 \equiv \frac{1}{nq-1}\sum_{k=1}^{nq}(P_k - P_{k-1} - \hat{\mu})^2 \tag{2}$$

$$\hat{\sigma}_c^2 \equiv \frac{1}{m}\sum_{k=q}^{nq}(P_k - P_{k-q} - q\hat{\mu})^2 \tag{3}$$

where

$$m = q(nq - q + 1)\left(1 - \frac{q}{nq}\right). \tag{4}$$

We define the variance ratio as follows:

$$VR(q) = \frac{\hat{\sigma}_c^2}{\hat{\sigma}_a^2}. \tag{5}$$

Each day, we study daily values of the Variance Ratio (5) with values of q ranging from 1 to 5, since in our case the interval between adjacent observations is 4 minutes. For these values of q, we can then safely use the VR test with high-frequency data in our context. In particular, Bianco and Renò (2006) show that the VR test can be implemented on high frequency data of stock index futures transactions, for time scales lower than 20 minutes, given the typical heteroskedasticity of this asset. This is in line with the robustness analysis of Deo and Richardson (2003).

We then compute $1,975$ daily values of the standardized variance ratio, indicated as $\widehat{VR}(q)$ (see appendix B), for $q = 1, \ldots, 5$. The top panel of Table 2 reports the number of significantly positive and negative variance ratios, for different confidence intervals. The positive violations are compatible with the null. The excess in negative violations can instead be ascribed to the bid-ask bounce effect, see the thorough discussion in Bianco and Renò (2006).

In order to quantify the daily serial correlation, we use the modification proposed by Chen and Deo (2006):

$$\widetilde{VR}(q) = \widehat{VR}(q)^{\beta} \tag{6}$$

where the exponent β is defined in appendix A.

This transformation has the advantage of adjusting for the non-normality of the variance ratio time series. Table 1 reports the sample moments of

$\widehat{VR}(q)$ and $\widetilde{VR}(q)$, showing that the transformed variables are much closer to a Normal distribution. The time series of daily modified variance ratios at $q = 1$ is shown in figure 1.

Given the high persistence in volatility, also the modified variance ratio is found to be highly persistent. We discuss further this point in Section 3.

We want to link serial correlation with volatility. Each day, in which we have $N = 101$ returns, we define volatility as

$$\sigma^2 = \sum_{k=1}^{N} r_k^2 \tag{7}$$

This is the well-known measure of realized variance, see Andersen et al. (2003)[1]. However, in what follows we argue that another variable plays a very special role, that is unexpected volatility. We know that volatility is highly foreseeable in financial markets, see Poon and Granger (2003) for a review, mainly because of its persistence property. Moreover, a simple linear model for realized volatility leads to fair forecasts, see e.g. Andersen et al. (2003); Corsi et al. (2001). We then assume that the market volatility is forecasted with the following linear model:

$$\log(\sigma_t^2) = \alpha + \sum_{i=1}^{L} \beta_i \log(\sigma_{t-i}^2) + \varepsilon_t. \tag{8}$$

Even if the model (8) is fairly simple, since it ignores long-memory and leverage effects, on the US stock index futures data it yields an R^2 of 66.2% at $L = 3$. We then define *unexpected volatility* as the residual of the above regression,

$$\sigma_{u,t}^L \equiv \hat{\varepsilon}_t. \tag{9}$$

We also define the predictable part of volatility, as:

$$\sigma_{p,t}^L \equiv \log(\sigma_t^2) - \sigma_{u,t}^L$$

By construction, lagged volatility at times $t-1, \ldots, t-L$ and unexpected volatility are orthogonal. Thus $\sigma_{p,t}^L$ and $\sigma_{u,t}^L$ are orthogonal as well.

It is clear that our definition of unexpected and predictable volatility is dependent on model (8), and in particular on the choice of L. In what follows, we drop the superscript L and we present our main results for $L = 3$;

[1]With $\Delta t = 4$ minutes there is no need to correct for microstructure effects, as in Bandi and Russell (2006).

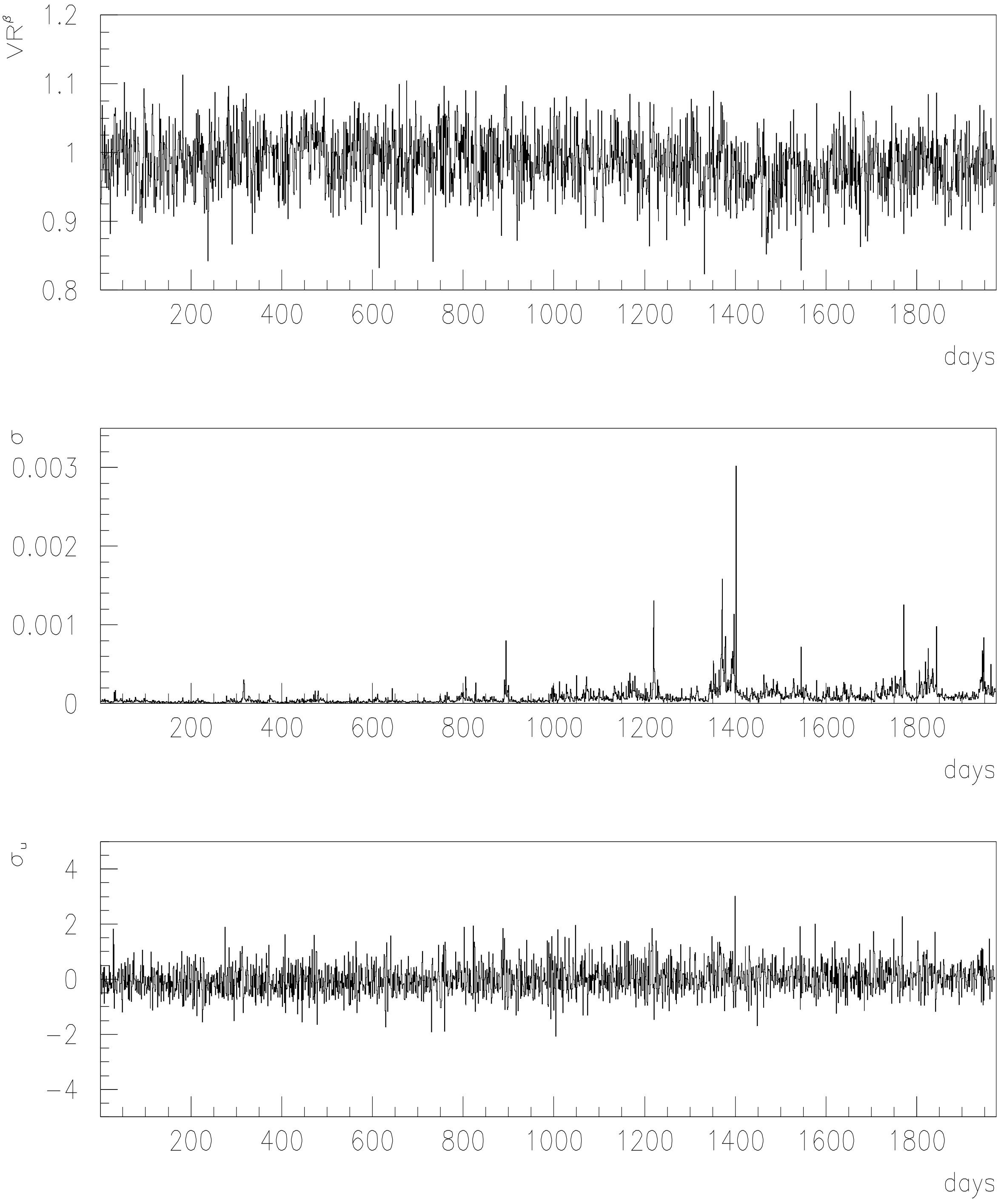

Figure 1: From top to bottom: the time series of $\widetilde{VR}(1)$, the daily realized volatility and the estimated unexpected volatility obtained using definition (9) with $L = 3$.

we subsequently show that they are robust to the choice of L. Related literature shows that including more complicated effects does not improve substantially the specification of model (8), see the extensive study of Hansen and Lunde (2005). Also nonlinear specifications, as those of Maheu and McCurdy (2002), have been found to yield forecast improvements which are marginal.

3 Results

Modified variance ratios are negatively autocorrelated by construction, since this feature is inherited by the serial correlation of the volatility. To show this, it is enough to simulate a long series of uncorrelated returns generated by a GARCH(1,1) process. If we compute the modified variance ratios on this series, we observe that it is serially correlated, see Bianco and Renò (2006).

This poses a specification problem, since we want to use the modified variance ratio as a dependent variable. To correct for this effect, in all our specifications we add lagged variance ratio regressors as additional explanatory variables. As an overall specification test for our regressions, we use the Ljung-Box test of residuals at lag 5 and we denote it by $Q(5)$.

We first estimate a model in which we include realized volatility as a regressor[2]:

$$\widetilde{VR}_t = \alpha + \sum_{i=1}^{4} \delta_i \widetilde{VR}_{t-i} + \beta \cdot \log(\sigma_t^2) + \varepsilon_t. \tag{10}$$

Results are in Table 3. We find that there is a positive and significant relation between volatility and standardized variance ratio, and the regression is well specified if we include enough autoregressive terms for the variance ratio, see the Ljung-Box statistics. This result is not entirely surprising. On a much smaller market (Italy), Bianco and Renò (2006) provide evidence of a positive relation between volatility and intraday serial correlation. This is different from what is typically found at daily level, where the correlation is found to be negative, according to the LeBaron effect (LeBaron, 1992; Sentana and Wadhwani, 1992). However, this result can be explained according to the model of reinforcement of opinions of Chan (1993). According to this model, serial correlation is introduced into data since, once an agent decides to buy, he or she observes more liquid substitutes and reinforces

[2] With a slight abuse of language, we do not distinguish between realized volatility and its natural logarithm.

his or her opinion according to the movements of the substitutes. This effect is stronger when volatility is higher, that is when the price moves more (or more rapidly). Thus, the Chan (1993) model posits a positive relation between volatility and intraday serial correlation which is at all reasonable. However, for the US market the Chan model is less tenable. Indeed, for the US it is unreasonable to look for a more liquid substitute. Thus, the effect of the reinforcement of opinions is likely to be milder. To better understand this effect, we compute the percentage of significant VRs as volatility increases. Significance can be assessed using asymptotic standard errors in the case of overlapping observations, which are provided by Richardson and Smith (1993), see Appendix B. The violations are reported in Table 2. On the contrary on what happens on the Italian market, where the percentage of positive violations increases when volatility increases, we find that this holds marginally for the US market, confirming our intuition that the mechanism of reinforcement of opinions is likely to play a minor role in a liquid market as the US stock index futures.

We then analyze the impact of unexpected volatility, computed according to the definition (9) with $L = 3$. We estimate the regression:

$$\widetilde{VR}_t = \alpha + \sum_{i=1}^{4} \delta_i \widetilde{VR}_{t-i} + \beta \cdot \sigma_{u,t} + \varepsilon_t. \tag{11}$$

Results are shown in Table 4. Unexpected volatility is found to be highly significant, and we obtain a good specification as measured by the Ljung-Box statistics, as long as we include enough lags of the variance ratio itself and for all the considered values of q. Thus, it is evident that unexpected volatility plays a crucial role in the emergence of intraday serial correlations, for all the considered time scales.

Most importantly, our results can be reconciled with the results in LeBaron (1992). To show this, we estimate the encompassing regression:

$$\widetilde{VR}_t = \alpha + \sum_{i=1}^{4} \delta_i \widetilde{VR}_{t-i} + \beta \cdot \sigma_{p,t} + \gamma \cdot \sigma_{u,t} + \varepsilon_t, \tag{12}$$

where both unexpected and predictable volatility are included as regressors. Results are displayed in Table 5 and indicate that, while volatility has been found to be significant in model (10), its predictable part is negatively related with intraday variance ratios, and its unexpected part is positively related. Indeed, LeBaron (1992) did not use realized measures of intraday variance, but he filtered the variance with a GARCH-like model, thus he considered only the predictable part, getting a negative relation. Since we are using a realized measure of volatility, we can decompose it into a predictable and unpredictable part, and we consistently find that the first has a negative impact on intraday serial correlation, while the second has a large positive impact. A negative relation between predictable volatility and intraday serial correlation could not be seen by Bianco and Renò (2006) in the Italian market, given the very low statistics (three years of data only). Thus, we conclude that unexpected volatility is the main source of intraday serial correlation, even if, at our knowledge, there is not an economic model explaining why the role of unexpected volatility is so important, since most economic models use total volatility.

4 Robustness analysis

In this Section, we show that the above reported results are robust to several assumptions.

First of all, we study the dependence of our results on the definition of unexpected volatility. We define it via the linear forecasting model (8), thus it is natural to ask whether the particular choice of L is relevant. For brevity's sake, we limit ourselves to the most interesting case of $q = 1$. We estimate equation (8) for several values of L, and for each value we get a different time series of unexpected and predictable volatility. We then re-estimate model (12), and we report estimates for each value of L, as well as the value of the R^2 of the regression (8). Results are reported in Table 8, and they show that the choice of L is not relevant. The R^2 of the forecasting regression with $L = 1$ is 61.31%, with $L = 20$ is 68.7%, and in this two extreme situations, as well as for intermediate values of L, predictable volatility has a significantly negative impact on the variance ratio, while unexpected volatility has a significantly positive impact. Thus, our result does not depend on the choice of L.

As a second robustness check, we look whether our results depend on the fact that we integrate over a day to compute the variance ratio, and if the effect is still present in morning returns as well as in returns before closure. To this purpose, we estimate the variance ratio using, respectively, the first 30 returns in the day and the last 30 returns in the day. Again, for brevity's sake, we limit our analysis to the case $q = 1$. Related studies on the microstructure of the market (Cao et al., 2004; Ellul et al., 2003) study the the difference in willing to trade large samples at different trading hours, which may affect price changes. Results are reported in Table 6 and 7 respectively. In this case the variance ratio measures are noisier, since we use less than one third of the observations used before. However, we find that the effect of positive impact of predictable volatility and negative impact of unexpected volatility is persistent; the t-ratios are just smaller, but they still indicate significance. Thus, these effects do not depend on the particular moment of the day.

Finally, we re-estimate our model using a larger spacing of equally spaced observations, namely $\Delta t = 10$ minutes. We fix the spacing and re-evaluate the regression (12). Also in this case, without loss of generality, the analysis is limited to the case $q = 1$. The results, reported in Table 9, confirm the preminent role of unexpected volatility with respect to the predictable part of volatility in the serial correlations formation. The signs of both expected and unexpected volatility remain the same as in the case of a smaller spacing, confirming the robustness of our analysis. The regression is well specified, as indicated by the value of $Q(5)$, even if we do not include auto-regressive terms. This may suggest the influence of the heteroskedasticity of the volatility on the VR to be milder at the considered spacing. However, it must be stressed that the number of daily data drops to 41, implying that the test loses its power and it is not reliable anymore, see Deo and Richardson (2003).

5 Conclusions

In this paper we study the impact of volatility on intraday serial correlation in the US stock index futures market, which is the most liquid market in the world. We exploit the availability of intraday data to measure volatility by means of realized variance, and intraday serial correlation by means of the variance ratio. To get time series closer to a normal distribution, we use logarithmic volatilities and suitably modified variance ratios.

We find that, in agreement with the economic theory, total volatility plays a minor role in the US market, since the mechanism of reinforcement of opinions postulated by Chan (1993) is less important in this market. We then use our realized measure to decompose volatility into its predictable and unpredictable parts, naming the latter *unexpected volatility*. We find that there is a positive and significant relation between unexpected volatility and intraday serial correlation, while we confirm the LeBaron effect: predictable volatility is negatively related to serial correlation.

This result can be important for the economic theory, since this could potentially reveal basic properties about the pricing formation mechanism. As far as we know, there are no economic theories explaining the stylized facts documented by our study, thus our results introduce a new challenge. However, we presume that the role of unexpected volatility is linked to the way information is spread in the market. In this respect, unexpected volatility could be potentially employed as a proxy for information asymmetry. Further research is needed to assess this conjecture.

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A The variance ratio correction

We use the results of Chen and Deo (2004, 2006), and we correct the estimated variance ratio according to formula (6), where the exponent β is given by:

$$\beta = 1 - \frac{2}{3} \frac{\left(\sum_{j=1}^{(n-1)/2} W_k(\lambda_j)\right)\left(\sum_{j=1}^{(n-1)/2} W_k^3(\lambda_j)\right)}{\left(\sum_{j=1}^{(n-1)/2} W_k^2(\lambda_j)\right)^2}, \quad (13)$$

where W_k is the Dirichlet kernel:

$$W_k(\lambda) = \frac{1}{k} \frac{\sin^2(k\lambda/2)}{\sin^2(\lambda/2)} \quad (14)$$

and $\lambda_j = 2\pi j/n$.

B Variance ratio asymptotic distribution

Under the null hypothesis of random walk, the asymptotic distribution of the statistics (5) is the following. Define:

$$\hat{\delta}_k = \frac{nq \sum_{j=k+1}^{nq} (P_j - P_{j-1} - \hat{\mu})^2 (P_{j-k} - P_{j-k-1} - \hat{\mu})^2}{\left[\sum_{j=1}^{nq} (P_j - P_{j-1} - \hat{\mu})^2\right]^2} \quad (15)$$

$$\hat{\theta}(q) = 4 \sum_{k=1}^{q-1} \left(1 - \frac{k}{q}\right)^2 \hat{\delta}_k. \quad (16)$$

Then we have:

$$\sqrt{nq}(\widehat{VR}(q) - 1) \sim N(0, \hat{\theta}), \quad (17)$$

The variance ratio test implemented here allows for heteroskedasticity, does not require the assumption of normality and in small samples it is more powerful than other tests, like the Ljung-Box statistics or the Dickey-Fuller unit root test, see Lo and MacKinlay (1989); Faust (1992); Cecchetti and Sang Lam (1994).

Variance Ratio (raw)

q	mean	variance	kurtosis -3	skewness
1	0.964	0.015	0.019	0.017
2	0.938	0.028	0.303	0.069
3	0.918	0.040	0.699	0.119
4	0.905	0.050	1.051	0.167
5	0.895	0.060	1.417	0.210

Variance Ratio (transformed)

q	mean	variance	kurtosis -3	skewness
1	0.986	0.002	0.126	-0.004
2	0.980	0.002	0.162	-0.002
3	0.978	0.002	0.238	0.001
4	0.975	0.002	0.147	0.004
5	0.973	0.003	0.391	0.004

Table 1: Sample moments of the variance ratio before (top part of the Table) and after (bottom part of the Table) the power transformation, for $q = 1, \ldots, 5$.

all $\sigma^2, 100\%$ of the sample						
q	$90\%^+$	$90\%^-$	$95\%^+$	$95\%^-$	$99\%^+$	$99\%^-$
1	0.064	0.209	0.034	0.120	0.004	0.034
2	0.057	0.202	0.029	0.103	0.005	0.021
3	0.051	0.191	0.025	0.093	0.006	0.013
4	0.047	0.173	0.024	0.065	0.006	0.003
5	0.044	0.157	0.024	0.053	0.006	0.001
$\sigma^2 > 3 \cdot 10^{-5}$, 68.5 % of the sample						
q	$90\%^+$	$90\%^-$	$95\%^+$	$95\%^-$	$99\%^+$	$99\%^-$
1	0.089	0.205	0.045	0.123	0.004	0.039
2	0.069	0.201	0.031	0.109	0.007	0.024
3	0.066	0.177	0.030	0.098	0.006	0.013
4	0.055	0.161	0.030	0.072	0.007	0.001
5	0.052	0.153	0.027	0.061	0.007	0.001
$\sigma^2 > 7.5 \cdot 10^{-5}$, 35.8 % of the sample						
q	$90\%^+$	$90\%^-$	$95\%^+$	$95\%^-$	$99\%^+$	$99\%^-$
1	0.088	0.212	0.041	0.127	0.007	0.048
2	0.059	0.213	0.027	0.113	0.007	0.021
3	0.055	0.185	0.021	0.100	0.007	0.013
4	0.042	0.154	0.025	0.068	0.007	0.001
5	0.042	0.145	0.025	0.058	0.008	0.000
$\sigma^2 > 1.4 \cdot 10^{-4}$, 15.8 % of the sample						
q	$90\%^+$	$90\%^-$	$95\%^+$	$95\%^-$	$99\%^+$	$99\%^-$
1	0.128	0.147	0.071	0.096	0.016	0.035
2	0.093	0.183	0.045	0.096	0.013	0.022
3	0.074	0.173	0.038	0.093	0.010	0.013
4	0.061	0.138	0.035	0.067	0.010	0.003
5	0.058	0.154	0.035	0.064	0.010	0.000
$\sigma^2 > 2 \cdot 10^{-4}$, 8.1 % of the sample						
q	$90\%^+$	$90\%^-$	$95\%^+$	$95\%^-$	$99\%^+$	$99\%^-$
1	0.151	0.132	0.094	0.082	0.025	0.038
2	0.119	0.164	0.069	0.101	0.013	0.019
3	0.101	0.151	0.057	0.094	0.013	0.013
4	0.069	0.132	0.038	0.069	0.013	0.006
5	0.063	0.138	0.038	0.069	0.013	0.000

Table 2: Percentage of significant positive and negative VR, for different significance levels (one-sided), on subsamples with growing daily volatility, see the top of each panel.

q	α	$\log(\sigma^2)$	$\widetilde{VR}_{t-1}$	$\widetilde{VR}_{t-2}$	$\widetilde{VR}_{t-3}$	$\widetilde{VR}_{t-4}$	Q(5)
1	1.025	0.004					168.00**
	(104.449)**	(4.010)**					
	0.860	0.004	0.168				55.35**
	(36.099)**	(4.086)**	(7.603)**				
	0.795	0.004	0.153	0.084			33.28**
	(26.882)**	(4.382)**	(6.857)**	(3.732)**			
	0.735	0.004	0.147	0.071	0.081		15.04*
	(21.720)**	(4.555)**	(6.573)**	(3.151)**	(3.608)**		
	0.694	0.005	0.143	0.069	0.074	0.055	6.21
	(18.711)**	(4.697)**	(6.399)**	(3.059)**	(3.279)**	(2.468)**	
2	1.007	0.003					103.11**
	(94.192)**	(2.515)**					
	0.871	0.003	0.141				35.52**
	(36.293)**	(2.718)**	(6.317)**				
	0.821	0.003	0.132	0.063			24.86**
	(27.383)**	(2.991)**	(5.883)**	(2.785)**			
	0.751	0.004	0.127	0.052	0.092		4.34
	(21.768)**	(3.347)**	(5.675)**	(2.282)*	(4.073)**		
	0.724	0.004	0.124	0.051	0.087	0.037	1.02
	(19.071)**	(3.483)**	(5.517)**	(2.254)*	(3.859)**	(1.659)*	
3	1.006	0.003					57.15**
	(92.925)**	(2.654)**					
	0.908	0.003	0.101				23.74**
	(37.485)**	(2.741)**	(4.526)**				
	0.876	0.003	0.097	0.039			19.41**
	(28.501)**	(2.871)**	(4.328)**	(1.715)*			
	0.810	0.003	0.095	0.031	0.080		3.17
	(22.634)**	(3.144)**	(4.220)**	(1.393)	(3.564)**		
	0.788	0.004	0.093	0.032	0.078	0.027	0.98
	(19.866)**	(3.236)**	(4.130)**	(1.418)	(3.459)**	(1.179)	
4	1.013	0.004					23.73**
	(90.440)**	(3.382)**					
	0.961	0.004	0.054				13.08*
	(39.030)**	(3.375)**	(2.386)**				
	0.944	0.004	0.052	0.019			11.70*
	(29.728)**	(3.398)**	(2.332)*	(0.837)			
	0.895	0.004	0.052	0.016	0.055		2.83
	(23.928)**	(3.545)**	(2.296)*	(0.717)	(2.446)**		
	0.881	0.004	0.051	0.018	0.054	0.015	1.57
	(21.019)**	(3.590)**	(2.265)*	(0.784)	(2.416)**	(0.671)	
5	1.016	0.004					6.56
	(86.080)**	(3.647)**					
	1.008	0.004	0.009				5.81
	(40.237)**	(3.636)**	(0.394)				
	1.006	0.004	0.009	0.002			5.96
	(30.735)**	(3.610)**	(0.388)	(0.070)			
	0.975	0.004	0.009	0.001	0.033		2.02
	(25.013)**	(3.685)**	(0.398)	(0.059)	(1.475)		
	0.966	0.004	0.009	0.003	0.033	0.007	1.44
	(21.940)**	(3.706)**	(0.381)	(0.147)	(1.480)	(0.314)	

Table 3: Estimates of model 10), for different values of q. * indicates 95% of confidence level, ** 99% of confidence level.

q	α	$\sigma_{u,t}$	$\widetilde{VR}_{t-1}$	$\widetilde{VR}_{t-2}$	$\widetilde{VR}_{t-3}$	$\widetilde{VR}_{t-4}$	Q(5)
1	0.986	0.023					180.96**
	(1089.39)**	(14.131)**					
	0.815	0.023	0.173				55.81**
	(39.107)**	(14.536)**	(8.198)**				
	0.716	0.025	0.153	0.120			24.79**
	(26.323)**	(15.278)**	(7.218)**	(5.601)**			
	0.648	0.025	0.147	0.107	0.089		8.27
	(20.522)**	(15.502)**	(6.910)**	(4.944)**	(4.209)**		
	0.607	0.025	0.143	0.104	0.082	0.054	1.46
	(17.406)**	(15.535)**	(6.729)**	(4.833)**	(3.849)**	(2.566)**	
2	0.981	0.025					134.04**
	(997.155)**	(14.342)**					
	0.842	0.025	0.141				48.55**
	(40.445)**	(14.545)**	(6.652)**				
	0.759	0.027	0.127	0.099			29.29**
	(27.536)**	(15.097)**	(5.982)**	(4.588)**			
	0.670	0.027	0.121	0.085	0.111		4.93
	(20.729)**	(15.553)**	(5.716)**	(3.963)**	(5.201)**		
	0.640	0.027	0.118	0.084	0.106	0.040	1.65
	(17.859)**	(15.576)**	(5.541)**	(3.913)**	(4.941)**	(1.858)*	
3	0.978	0.026					73.73**
	(981.514)**	(14.282)**					
	0.882	0.026	0.098				32.15**
	(42.210)**	(14.325)**	(4.594)**				
	0.821	0.026	0.091	0.069			23.94**
	(29.162)**	(14.632)**	(4.256)**	(3.205)**			
	0.736	0.027	0.088	0.060	0.099		4.56
	(21.992)**	(15.018)**	(4.116)**	(2.803)**	(4.638)**		
	0.710	0.027	0.085	0.060	0.096	0.031	1.87
	(18.948)**	(15.035)**	(4.004)**	(2.811)**	(4.495)**	(1.468)	
4	0.975	0.027					28.88**
	(946.590)**	(14.391)**					
	0.928	0.027	0.049				16.95**
	(44.361)**	(14.368)**	(2.271)*				
	0.889	0.027	0.046	0.043			14.22*
	(30.846)**	(14.501)**	(2.158)*	(1.991)*			
	0.819	0.028	0.045	0.039	0.076		3.36
	(23.545)**	(14.785)**	(2.108)*	(1.833)*	(3.546)**		
	0.799	0.028	0.044	0.041	0.075	0.022	1.49
	(20.252)**	(14.786)**	(2.059)*	(1.888)*	(3.494)**	(1.016)	
5	0.973	0.027					8.31
	(893.361)**	(13.927)**					
	0.971	0.027	0.002				8.05
	(46.289)**	(13.916)**	(0.109)				
	0.950	0.027	0.002	0.022			7.47
	(32.197)**	(13.954)**	(0.096)	(1.036)			
	0.897	0.028	0.002	0.022	0.054		2.21
	(24.838)**	(14.136)**	(0.095)	(1.028)	(2.524)**		
	0.882	0.028	0.002	0.024	0.054	0.014	1.47
	(21.313)**	(14.122)**	(0.070)	(1.108)	(2.523)**	(0.649)	

Table 4: Estimates of model (11), for different values of q. * indicates 95% of confidence level, ** 99% of confidence level.

q	α	$\sigma_{p,t}$	$\sigma_{u,t}$	$\widetilde{VR}_{t-1}$	$\widetilde{VR}_{t-2}$	$\widetilde{VR}_{t-3}$	$\widetilde{VR}_{t-4}$	Q(5)
1	0.929	-0.006	0.023					138.83**
	(81.087)**	(-5.029)**	(14.218)**					
	0.756	-0.006	0.023	0.174				34.92**
	(31.969)**	(-5.191)**	(14.633)**	(8.299)**				
	0.658	-0.006	0.025	0.155	0.119			14.79*
	(22.478)**	(-5.198)**	(15.375)**	(7.318)**	(5.607)**			
	0.594	-0.006	0.025	0.148	0.107	0.086		5.92
	(17.898)**	(-5.096)**	(15.590)**	(7.015)**	(4.967)**	(4.084)**		
	0.558	-0.006	0.025	0.145	0.104	0.080	0.049	2.97
	(15.480)**	(-4.940)**	(15.616)**	(6.847)**	(4.868)**	(3.763)**	(2.318)*	
2	0.893	-0.009	0.025					81.18**
	(72.221)**	(-7.109)**	(14.521)**					
	0.761	-0.009	0.025	0.137				24.24**
	(32.257)**	(-6.980)**	(14.719)**	(6.514)**				
	0.684	-0.008	0.026	0.124	0.094			18.04**
	(23.321)**	(-6.853)**	(15.238)**	(5.874)**	(4.398)**			
	0.604	-0.008	0.027	0.118	0.081	0.103		7.28
	(18.033)**	(-6.629)**	(15.662)**	(5.627)**	(3.814)**	(4.906)**		
	0.584	-0.008	0.027	0.116	0.081	0.100	0.028	7.37
	(15.983)**	(-6.468)**	(15.667)**	(5.497)**	(3.786)**	(4.718)**	(1.338)	
3	0.892	-0.009	0.026					41.41**
	(71.132)**	(-6.895)**	(14.450)**					
	0.801	-0.009	0.026	0.094				15.11**
	(33.594)**	(-6.809)**	(14.491)**	(4.467)**				
	0.745	-0.008	0.026	0.088	0.065			14.77*
	(24.778)**	(-6.731)**	(14.774)**	(4.147)**	(3.042)**			
	0.667	-0.008	0.027	0.085	0.057	0.093		6.75
	(19.188)**	(-6.575)**	(15.137)**	(4.017)**	(2.663)**	(4.413)**		
	0.650	-0.008	0.027	0.083	0.057	0.092	0.022	6.80
	(16.982)**	(-6.457)**	(15.136)**	(3.939)**	(2.680)**	(4.314)**	(1.023)	
4	0.897	-0.008	0.027					15.18**
	(69.021)**	(-6.016)**	(14.519)**					
	0.851	-0.008	0.027	0.048				7.78
	(34.942)**	(-6.004)**	(14.496)**	(2.244)*				
	0.813	-0.008	0.027	0.045	0.041			8.53
	(26.078)**	(-5.986)**	(14.623)**	(2.134)*	(1.939)*			
	0.747	-0.008	0.027	0.044	0.038	0.074		4.54
	(20.418)**	(-5.940)**	(14.899)**	(2.085)*	(1.786)*	(3.471)**		
	0.733	-0.008	0.027	0.044	0.039	0.073	0.015	4.48
	(18.010)**	(-5.867)**	(14.888)**	(2.051)*	(1.840)*	(3.434)**	(0.724)	
5	0.900	-0.007	0.027					4.85
	(65.341)**	(-5.339)**	(14.024)**					
	0.898	-0.007	0.027	0.003				4.71
	(35.935)**	(-5.338)**	(14.013)**	(0.120)				
	0.876	-0.007	0.027	0.002	0.022			5.38
	(27.059)**	(-5.339)**	(14.052)**	(0.107)	(1.046)			
	0.824	-0.007	0.028	0.002	0.022	0.054		3.86
	(21.464)**	(-5.336)**	(14.232)**	(0.107)	(1.038)	(2.520)**		
	0.814	-0.007	0.028	0.002	0.024	0.054	0.010	4.06
	(18.885)**	(-5.286)**	(14.211)**	(0.087)	(1.114)	(2.521)**	(0.455)	

Table 5: Estimates of model (12), for different values of q. * indicates 95% of confidence level, ** 99% of confidence level.

α	$\sigma_{p,t}$	$\sigma_{u,t}$	$\widetilde{VR}_{t-1}$	$\widetilde{VR}_{t-2}$	$\widetilde{VR}_{t-3}$	$\widetilde{VR}_{t-4}$	Q(5)
0.936	-0.005	0.022					40.83**
(50.279)**	(-2.814)**	(8.485)**					
0.825	-0.005	0.023	0.111				14.76*
(28.819)**	(-2.876)**	(8.636)**	(5.047)**				
0.784	-0.005	0.023	0.106	0.047			11.35*
(22.557)**	(-2.882)**	(8.831)**	(4.800)**	(2.130)*			
0.744	-0.005	0.023	0.105	0.042	0.047		6.48
(18.904)**	(-2.829)**	(8.880)**	(4.749)**	(1.904)*	(2.148)*		
0.698	-0.005	0.023	0.102	0.041	0.041	0.058	1.42
(16.250)**	(-2.740)**	(8.857)**	(4.631)**	(1.842)*	(1.848)*	(2.640)**	

Table 6: Estimates of model (12), for different values of q, when using only **morning** returns (first 30) to estimate modified variance ratios. * indicates 95% of confidence level, ** 99% of confidence level.

α	$\sigma_{p,t}$	$\sigma_{u,t}$	$\widetilde{VR}_{t-1}$	$\widetilde{VR}_{t-2}$	$\widetilde{VR}_{t-3}$	$\widetilde{VR}_{t-4}$	Q(5)
0.925	-0.006	0.017					33.55^{**}
(49.478)**	(-3.234)**	(6.545)**					
0.880	-0.006	0.017	0.046				26.29^{**}
(30.785)**	(-3.209)**	(6.560)**	(2.066)*				
0.806	-0.006	0.018	0.042	0.081			11.56^{*}
(22.925)**	(-3.157)**	(6.814)**	(1.901)*	(3.648)**			
0.747	-0.006	0.018	0.037	0.078	0.068		4.02
(18.743)**	(-3.080)**	(6.971)**	(1.681)*	(3.536)**	(3.077)**		
0.743	-0.006	0.018	0.038	0.081	0.066	0.005	4.08
(17.063)**	(-3.035)**	(6.939)**	(1.694)*	(3.629)**	(2.994)**	(0.204)	

Table 7: Estimates of model (12), for different values of q, when using only **evening** returns (last 30) to estimate modified variance ratios. * indicates 95% of confidence level, ** 99% of confidence level.

L	α	$\sigma^L_{p,t}$	$\sigma^L_{u,t}$	$\widetilde{VR}_{t-1}$	$\widetilde{VR}_{t-2}$	$\widetilde{VR}_{t-3}$	$\widetilde{VR}_{t-4}$	Q(5)	R^2
1	0.554	-0.006	0.021	0.186	0.096	0.049	0.048	3.408	0.613
	(15.061)**	(-4.895)**	(14.023)**	(8.642)**	(4.445)**	(2.249)*	(2.258)*		
2	0.560	-0.005	0.023	0.152	0.123	0.061	0.041	4.682	0.648
	(15.397)**	(-4.722)**	(14.807)**	(7.164)**	(5.644)**	(2.855)**	(1.934)*		
3	0.558	-0.006	0.025	0.145	0.104	0.080	0.049	2.969	0.662
	(15.480)**	(-4.940)**	(15.616)**	(6.847)**	(4.868)**	(3.763)**	(2.318)*		
4	0.566	-0.006	0.025	0.138	0.101	0.067	0.064	3.229	0.669
	(15.787)**	(-4.953)**	(15.864)**	(6.532)**	(4.732)**	(3.163)**	(3.051)**		
5	0.584	-0.005	0.026	0.135	0.097	0.066	0.055	1.920	0.674
	(16.400)**	(-4.897)**	(15.899)**	(6.398)**	(4.532)**	(3.086)**	(2.625)**		
6	0.591	-0.006	0.026	0.134	0.096	0.063	0.054	1.828	0.676
	(16.637)**	(-4.972)**	(16.045)**	(6.337)**	(4.474)**	(2.957)**	(2.542)**		
10	0.611	-0.006	0.027	0.129	0.091	0.058	0.046	1.310	0.682
	(17.343)**	(-5.097)**	(16.364)**	(6.132)**	(4.262)**	(2.747)**	(2.174)*		
20	0.623	-0.006	0.027	0.127	0.086	0.056	0.041	1.706	0.687
	(17.774)**	(-5.281)**	(16.664)**	(6.025)**	(4.036)**	(2.621)**	(1.947)*		

Table 8: Estimates of model (12), for different values of L and $q = 1$. The column R^2 indicates the R^2 of the regression (8). * indicates 95% of confidence level, ** 99% of confidence level.

q	α	$\sigma_{p,t}$	$\sigma_{u,t}$	$\widetilde{VR}_{t-1}$	$\widetilde{VR}_{t-2}$	$\widetilde{VR}_{t-3}$	$\widetilde{VR}_{t-4}$	Q(5)
1	0.963	-0.002	0.013					0.67
	(58.265)**	(-1.071)	(5.426)**					
	0.979	-0.002	0.013	-0.016				0.89
	(35.222)**	(-1.054)	(5.445)**	(-0.709)				
	0.973	-0.002	0.013	-0.016	0.006			0.88
	(27.154)**	(-1.060)	(5.443)**	(-0.705)	(0.269)			
	0.962	-0.002	0.013	-0.016	0.006	0.010		0.95
	(22.703)**	(-1.074)	(5.458)**	(-0.708)	(0.275)	(0.451)		
	0.953	-0.002	0.013	-0.016	0.006	0.010	0.010	0.64
	(19.878)**	(-1.083)	(5.472)**	(-0.713)	(0.268)	(0.452)	(0.448)	

Table 9: Estimate of (12) with spacing $\Delta t = 10$, for $q = 1$. * indicates 95% of confidence level, ** 99% of confidence level.